Module :- 3

Design for shafts, keys & couplings

Shaft :- shaft is a rotating member in general has a circular cross-section & it is used to transmit the power.

shaft may be either solid (or) hallow shaft.

Types of shaft

1) **Transmission shaft** :- A shaft carries power transmitting elements ex:- Gears, pulley etc.

2) **machine shaft** :- A shaft is used in the machine ex:- cam shaft, crank shaft

Design criterian

1) Design based on strength

2) Design based on stiffness

1) Design based on strength

The load acts on the shaft are axial, bending & torsional load.

i) **Axial load**

For solid shaft

$$\sigma_d = \frac{P}{A} = \frac{P}{\pi d^2/4}$$

For hallow shaft

$$\sigma_d = \frac{P}{A} = \frac{P}{\pi (d_o^2 - d_i^2)/4}$$

ii) **Bending load**

For solid shaft

$$\therefore \boxed{\sigma_b = \frac{32M}{\pi d^3}}$$

For hallow shaft

$$\sigma_b = \frac{M \times y}{I} = \frac{M \times (d_o/2)}{\pi/64 (d_o^4 - d_i^4)}$$

$$= \frac{M \times (d_o/2)}{\frac{\pi}{32} d_o^4 \left(1 - \left(\frac{d_i}{d_o}\right)^4\right)}$$

where $k = \frac{d_i}{d_o}$

$$\therefore \boxed{\sigma_b = \frac{32 M}{\pi d_o^3 (1 - k^4)}}$$

iii) Torsional load

For solid shaft

$$\tau = \frac{16 T}{\pi d^3}$$

For hallow shaft

$$\tau = \frac{16 T}{\pi d_o^3 (1 - k^4)}$$

where $k = \frac{d_i}{d_o}$

when it is subjected to combine loading the design equations are obtained with the help of max. ~~Pericipe~~ Principal stress theory & max. shear stress theory.

i) Based on max. Principal stress theory

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

If Bending $\sigma = \sigma_b$

$$\sigma_1 = \left[\frac{16 M_b}{\pi d^3}\right] + \sqrt{\left[\frac{16 M_b}{\pi d^3}\right]^2 + \left[\frac{16 M_t}{\pi d^3}\right]^2}$$

$$\sigma_1 = \left[\frac{16 M_b}{\pi d^3}\right] + \frac{16}{\pi d^3} \sqrt{(M_b)^2 + (M_t)^2}$$

$$\sigma_1 = \frac{16}{\pi d^3} \left[M_b + \sqrt{M_b^2 + M_t^2}\right] = \frac{\sigma_y}{n}$$

wkT

$$n = \frac{\sigma_y}{\sigma_{max}} = \frac{\sigma_y}{\sigma_1}$$

(or)

$$d^3 = \frac{16}{\pi \sigma_1} \left[M_b + \sqrt{M_b^2 + M_t^2}\right]$$

ii) Based on max shear stress theory.

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\therefore \tau_{max} = \frac{16}{\pi d^3}\sqrt{(M_b)^2 + (M_t)^2} = \frac{\tau_y}{n}$$

WKT $n = \frac{\tau_y}{\tau_{max}}$

(or)

$$d^3 = \frac{16}{\pi \tau_{max}} \cdot \sqrt{(M_b)^2 + (M_t)^2}$$

Problems

1) A solid shaft transmit power of 20kW at 500rpm. If maximum torque exceeds mean torque by 30% find the diameter of the shaft. The shaft is subjected to a twist of 1° in length of 2m. Take modulus of rigidity as 0.84×10^5 MPa.

Sol^n [we must & should put value of P in kW]

$P = 20\times10^3 W = 20kW$

$N = 500 rpm$

$T_{max} = 30\% \cdot T_{mean}$

$d = ?$

$\theta = 1°$

$L = 2m = 2\times10^3 mm$

$G = 0.84\times10^5 N/mm^2$

Torsional equal

$$\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r} \rightsquigarrow eq^n\ 1.3(b), pg\ 3$$

$$\frac{T_{max}}{J} = \frac{G\theta}{L}$$

To find the T_{mean}

$$T_{mean} = \frac{9.55\times10^6 \times P}{n} \rightsquigarrow [eq\ 3.3\ (a), pg\ si$$

$$T_{mean} = \frac{9.55\times10^6 \times 20}{500}$$

$$\boxed{T_{mean} = 382\times10^3\ N\text{-}mm}$$

$T_{max} = 1.3\times T_{mean}$

$T_{max} = 1.3 \times 382\times10^3$

$$\boxed{T_{max} = 496.6\times10^3\ N\text{-}mm}$$

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$$\frac{T_{max}}{J} = \frac{G\theta}{l}$$ θ should be in radians

$$\frac{T_{max}}{\pi d^4/32} = \frac{G \times \theta \times \pi/180}{l}$$

$$\cancel{\pi d^4 = \frac{T_{max} \times 32 \times 180 \times l}{G \times \theta \times \pi}}$$

$$\cancel{d^4 = \frac{T_{max} \times 32 \times 180 \times l}{G \times \theta \times \pi^2}}$$

$$\cancel{d^4 = \frac{(496.6 \times 10^3) \times 32 \times 180 \times 2 \times 10^3}{0.84 \times 10^5 \times 1^\circ \times \pi^2}}$$

$$\cancel{\boxed{d = 51.2530\,mm}}$$

$$\frac{\pi d^4}{32} = \frac{T_{max}}{\frac{G \times \theta \times \pi/180}{l}}$$

$$\frac{\pi d^4}{32} = \frac{T_{max} \times l \times 180}{G \times \theta \times \pi}$$

$$d^4 = \frac{32 \times T_{max} \times l \times 180}{G \times \theta \times \pi^2}$$

$$d^4 = \frac{32 \times (496.6 \times 10^3) \times 180 \times 2 \times 10^3}{0.84 \times 10^5 \times 10 \times \pi^2}$$

$$\boxed{d = 51.2530\,mm}$$

2) Determine the diameter of a solid shaft to transmit a Power of 88.2kW at a speed of 2000 rpm. If the angle of twist per meter length of shaft not exceed 0.1°. the modulus of rigidity is 84.04 MPa.

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Soln

$d = ?$

$P = 88.2 kW$

$N = 2000 rpm$

$L = 1m = 1\times10^3 mm$

$\theta = 0.1^\circ$

$G = 824.04 N/mm^2$

~~P = 88.2 KW~~

Torsional equal

$$T = \frac{9.55\times10^6\times P}{N} \rightarrow eq^n\ 3.3(a), Pg 50$$

$$T = \frac{9.55\times10^6\times 88.2}{2000}$$

$$\boxed{T = 421.155\times10^3 N\text{-}mm}$$

$$\frac{T}{J} = \frac{G\theta}{l}$$

$$\frac{T}{\pi d^4/32} = \frac{G\times\theta\times\pi/180}{l}$$

$$\frac{\pi d^4}{32} = \frac{T\times l}{G\times\theta\times\pi/180}$$

$$d^4 = \frac{T\times l\times 32\times 180}{G\times\theta\times\pi^2}$$

$$d^4 = \frac{421.155\times10^3\times1\times32\times180\times10^3}{824.04\times0.1\times(\pi^2)}$$

$$\boxed{\cancel{d = 41.35 mm}}$$

$\therefore d = 233.69\ mm$

3) A solid shaft transmitting 1MW at 240rpm. determine the diameter of the shaft. if maximum permissible torque exceeds mean torque by 20%. take maximum allowable shear stres as 60MPa.

Soln

$P = 1MW = 1000 kW$

$N = 240 rpm$

$d = ?$

$T_{max} = 1.2\times T_{mean}$

$\tau_{max} = 60 N/mm^2$

$$T = \frac{9.55\times10^6\times P}{N}\quad eq^n\ 3.3(a), Pg 5$$

$$T = \frac{9.55\times10^6\times1000}{240}$$

$$\boxed{T_{mean} = 39.7916\times10^6 N\text{-}mm}$$

$T_{max} = 1.2 \times T_{mean}$

$$\boxed{T'_{max} = 47.75 \times 10^6 \text{ N-mm}}$$

Torsional equation

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}$$

$$\frac{T_{max}}{\frac{\pi d^4}{32}} = \frac{\tau}{\cancel{d}/2}$$

$$\frac{T_{max} \times 32}{\pi d^3} = 2\tau$$

$$d^3 = \frac{T_{max} \times 32}{\pi \times 2 \times \tau} = \frac{47.75 \times 10^6 \times 32}{\pi \times 2 \times 60}$$

$$\boxed{d = 159.44 \text{ mm}}$$

ASME Code for design of transmission shafting

a) According to ~~sh~~ max. normal stress theory

For hallow shaft

$$d_o = \left[\frac{16}{\pi \sigma_{max}} \left(C_m M + \sqrt{(C_m M)^2 + (C_t T)^2}\right) \times \left(\frac{1}{1-k^4}\right)\right]^{1/3}$$

$\rightsquigarrow$ [eqn 3.6(a), Pg 51]

b) According to maximum shear stress theory

$$d_o = \left[\frac{16}{\pi \tau_{max}} \left(\sqrt{(C_m M)^2 + (C_t T)^2}\right) \times \left(\frac{1}{1-k^4}\right)\right]^{1/3}$$

$\rightsquigarrow$ [eqn 3.6 (b), Pg 51]

The out side diameter of hollow shaft subjected to an axial load F in addition to the torsional & bending loads.

a) According to max. normal stress theory.

$$d_o = \left[\frac{16}{\pi \sigma_{max}}\left\{\left(C_m M + \frac{\alpha F d_o (1+k^2)}{8}\right) + \sqrt{\left(C_m M + \frac{\alpha F d_o (1+k^2)}{8}\right)^2 + (C_t T)^2}\right\} \times \left(\frac{1}{1-k^4}\right)\right]^{1/3}$$

~> [eqⁿ 3.8 (a) Pg 51]

b) According to max. shear stress theory

$$d_o = \left[\frac{16}{\pi \tau_{max}}\left\{\sqrt{\left(C_m M + \frac{\alpha F d_o (1+k^2)}{8}\right)^2 + (C_t T)^2}\right\} \times \left(\frac{1}{1-k^4}\right)\right]^{1/3}$$

~> [eqⁿ 3.8 (b) Pg 51]

In question paper

$C_m = k_m$
$C_t = k_t$ } [The values are available in Table 3.1 in pg 56]

Problem

1) A line shaft rotating at 500 rpm is to transmit 600 kW the allowable shear stress for the material is 42 MPa. If the shaft carries a central load of 900 N & its simply supported b/w the bearings 3 m apart. determine the diameter of the shaft ~~to~~ the maximum tensile (or) compressive stress ~~or~~ is not to exceed 50 MPa.

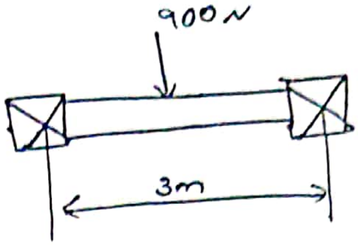
Soln: -

$n = 500\,rpm$, $P = 600\,kW$

$\tau_{max} = 42\,MPa$, $W = 900\,N$

$L = 3\,m$, $d = ?$

$\sigma_{max} = 50\,MPa$



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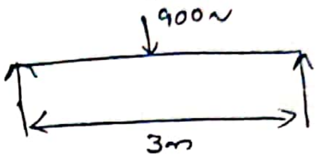


$$\text{Torque, } T = \frac{9.55\times10^6 \times P}{n} \rightarrow eq^n\ 3.3(a),\ pg\ 50$$

$$T = \frac{9.55\times10^6 \times 600}{500}$$

$$\therefore \boxed{T = 11.46\times10^6\ N\text{-}mm}$$

For simply supported beam → pg 15

$$M_{max} = \frac{WL}{4} = \frac{900\times3\times10^3}{4}$$

$$\boxed{M_{max} = 675\times10^3\ N\text{-}mm}$$

According to ASME code

For solid shaft)

$k=0$

a) max. normal stress theory

$$d^3 = \left[\frac{16}{\pi\sigma_{max}}\left(C_m M + \sqrt{(C_m M)^2 + (C_t T)^2}\right)\right]\cancel{\left(\frac{1}{1-k^4}\right)}$$

$\sigma_{max} = 50\,MPa$

Assume, steady load

$C_m = 1.5$

$C_t = 1.0$

$$d^3 = \left[\frac{16}{\pi\times50}\left(1.5\times675\times10^3 + \sqrt{(1.5\times675\times10^3)^2 + (1\times11.46\times10^6)^2}\right)\right]$$

$\therefore \boxed{d_o = 108.4347\,mm}$

b) max. shear stress theory

$$d^3 = \left[\frac{16}{\pi \tau_{max}} \sqrt{(C_m M)^2 + (C_t T)^2}\right]$$

$\tau_{max} = 42\,MPa$

assume steady load

$C_m = 1.5$

$C_t = 1.0$

$$d^3 = \left[\frac{16}{\pi \times 42} \sqrt{(1.5 \times 675 \times 10^3)^2 + (1 \times 11.46 \times 10^6)^2}\right]$$

$\therefore \boxed{d = 111.7372\,mm}$

The ~~diarnel~~ recommended diameter is 111.7372 mm

2) Determine the diameter of an hallow shaft to sustain a twisting moment of 600 N-m & a bending moment of 750 N-m limitting the angle of twist to 0.5° per meter length. The diameter ratio is 0.6. Take $C_m = 1.75$, $C_b = 1.25$ & $G = 80 \times 10^3\,MPa$.

Soln :-

$T = 600 \times 10^3\,N\text{-}mm$

$M = 750 \times 10^3\,N\text{-}mm$

$\theta = 0.5°$

$L = 1m = 1000\,mm$

$\frac{d_i}{d_o} = k = 0.6$

$C_m = 1.75$

$C_b = 1.25$

$G = 80 \times 10^3\,N/mm^2$

~~$\sigma_{max} = \frac{M \times y}{I}$~~

~~$\sigma_{bmax} = \frac{32M}{\pi d_o^3 (1 - k^4)}$~~

~~$\sigma_{max} = \frac{32 \times 750 \times 10^3}{\pi \times d_o^3 (1 - 0.6^4)}$~~

~~$\sigma_{max} = \frac{8.77692 \times 10^6}{d_o^3}$~~

Imp Note :-

- The ASME code recommends that value of τ_{max} to be used for commercial steel ~~shaft~~ shaft (ordinary steel shaft)

$\tau_{max} = 55 MPa$ [without keyway]

$\sigma_{max} = 110 MPa$ [without keyway]

- The values should be reduced by 25% if key way is present

$\tau_{max} = 0.75 \times 55 = \underline{41.25 MPa}$

$\sigma_{max} = 110 MPa = $

$= 0.75 \times 110$

$\therefore \sigma_{max} = \underline{82.5 MPa}$

- The ASME code recommends that the value of τ_{max} to be used for steel purchased under definite specification is

$$\left.\begin{aligned} \tau_{max} &= 0.3\,\sigma_y \\ \tau_{max} &= 0.18\,\sigma_u \end{aligned}\right\} \rightarrow \text{minimum of these two}$$

||'y

$$\left.\begin{aligned} \sigma_{max} &= 0.6\,\sigma_y \\ \sigma_{max} &= 0.36\,\sigma_u \end{aligned}\right\} \text{minimum of these two}$$

The values should be reduced by 25% if keyways are present.

Based on ASME codes

$\tau_{max} = 55 MPa$

$\sigma_{max} = 110 MPa$

a) max. normal stress theory.

$$d_o^3 = \left[\frac{16}{\pi \sigma_{max}}\left(C_m M + \sqrt{(C_m M)^2 + (C_t T)^2}\right) \times \left(\frac{1}{1-k^4}\right)\right] \rightarrow \text{eq } 3.6(a), \text{Pg } 31$$

$$= \left[\frac{16}{\pi \times 110}\left((1.75 \times 750 \times 10^3) + \sqrt{(1.75 \times 750 \times 10^3)^2 + (1.25 \times 600 \times 10^3)^2}\right) \times \left(\frac{1}{1-(0.6^4)}\right)\right]$$

$$\boxed{d_o = 53.159\,mm}$$

max. shear stress theory.

$$d_o^3 = \left[\frac{16}{\pi \tau_{max}}\left(\sqrt{(C_m M)^2 + (C_t T)^2}\right) \times \left(\frac{1}{1-(0.6^4)}\right)\right]$$

$$= \frac{16}{\pi \times 55} \times \left(\sqrt{(1.75 \times 750 \times 10^3)^2 + (1.25 \times 600 \times 10^3)^2}\right) \times \left(\frac{1}{1-(0.6^4)}\right)$$

$$\boxed{d_o = 54.3812\,mm}$$

the recommended outter diameter $d_o = 54.3812\,mm$

WKT $k = \frac{d_i}{d_o} = 0.6$

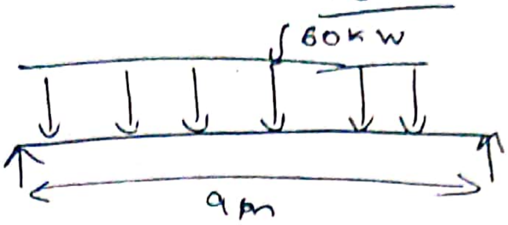
$d_i = 0.6 \times 54.3812$

$$\boxed{d_i = 32.628\,mm}$$

3) The hollow shaft of diameter 300mm & 400mm is mounted on two bearings, 9m apart & transmit 90kW at 1800 rpm. The shaft weights 60kN. Determine the stress induced in the shaft take $C_m = C_t = 1.5$.

Sol^n

Pg 15



$d_i = 300mm$

$d_o = 400mm$

$L = 9m = 9000mm$

$P = 90kw$

$n = 1800rpm$

$w = 60kN$

$\sigma_{max} = ?$

$\tau_{max} = ?$

$C_m = C_t = 1.5$

$$k = \frac{d_i}{d_o} = \frac{300}{400} = 0.75$$

considering the weight to be similar to UDL

max. Bending moment,

$$M = \frac{wL}{8} = \frac{(60\times10^3)(9000)}{8}$$

$$\boxed{M = 67.5\times10^6 \ N\text{-}mm}$$

Torque

$$T = \frac{9.55\times10^6 \times P}{n} \rightarrow eq^n \ 3.3(a), Pg\ 50$$

$$T = \frac{9.55\times10^6 \times 90}{1800}$$

$$\boxed{T = 477.5\times10^3 \ N\text{-}mm}$$

a) According to maximum normal stress theory

→ eq 3.6(a) Pg 5

$$d_o^3 = \frac{16}{\pi\sigma_{max}}\left(C_m M + \sqrt{(C_m M)^2 + (C_t T)^2}\right)\times\left(\frac{1}{1-k^4}\right)$$

$$\sigma_{max} = \frac{16}{\pi d_o^3}\left(C_m M + \sqrt{(C_m M)^2 + (C_t T)^2}\right)\times\frac{1}{(1-k^4)}$$

$$\sigma_{max} = \frac{16}{\pi\times(400^3)}\left(1.5\times67.5\times10^6 + \sqrt{(1.5\times67.5\times10^6)^2 + (1.5\times477.5\times10^3)^2}\right)\times\frac{1}{(1-(0.75^4))}$$

$$\underline{\sigma_{max} = 23.5734 \ N/mm^2}$$

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b) max. shear stress theory.

$$d_o^3 = \left[\frac{16}{\pi \tau_{max}}\left(\sqrt{(C_m M)^2 + (C_t T)^2}\right)\left(\frac{1}{1-k^4}\right)\right] \rightarrow \text{eq}^n\ 3.6(b)\ \text{Pg } 51$$

$$\tau_{max} = \frac{16}{\pi (400)^3}\left(\sqrt{(1.5\times 67.5\times 10^6)^2 + (1.5\times 477.5\times 10^3)^2}\right)\left(\frac{1}{1-(0.75^4)}\right)$$

$$\therefore \tau_{max} = 11.7868\ N/mm^2$$

4) A hollow shaft 500mm outside diameter & 300mm inside diameter is used to drive a propeller of a marine vessel. the shaft is mounted on bearings 6m apart & transmits 60,000 kw power at 150rpm. the maximum axial propeller thrust is 500kN & the shaft weighs 70kN determine i) maximum shear stress developed in the shaft ii) angular twist b/w the bearing take G = 84GPa

Data.

$d_o = 500mm$

$d_i = 300\ mm$

$L = 6m$

$P = 60000\ kW$

$n = 150\ rpm$

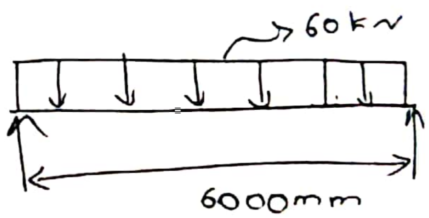
$W = 70kN$

$\tau_{max} = ?$

$G = 84GPa$

$\theta = ?$

$F = 500kN$



According to ASME code of max shear stress theory.

$$d_o = \left[\frac{16}{\pi \tau_{max}}\left\{\sqrt{\left(C_m M + \frac{\alpha F d_o (1+k^2)}{8}\right)^2 + (C_t T)^2}\right\}\left(\frac{1}{1-k^4}\right)\right.$$

$\rightarrow$ [eq^n 3.8(b) Pg 51]

assuming load is uniformly distributed

for simply supported beam, carrying UDL.

$$M_{max} = \frac{WL}{8} = \frac{70\times 10^3 \times 6000}{8} = 52.5\times 10^6\ N\text{-}mm$$

$$\text{Torque, } T = \frac{9.55\times 10^6 \times P}{n} \rightarrow [\text{eq}^n\ 3.3(a),\ \text{Pg } 50]$$

$$= \frac{9.55 \times 10^6 \times 60000}{150}$$

$$\therefore T = 3.82 \times 10^9 \, N\text{-}mm$$

Radius of gyration, $k = \sqrt{\frac{I}{A}}$

moment of Inertia, $I = \frac{\pi}{64}(d_o^4 - d_i^4)$

$$I = \frac{\pi}{64}\left[(500)^4 - (300)^4\right]$$

$$I = 2.670 \times 10^9 \, mm^4$$

Area of cross-section

$$A = \pi/4(d_o^2 - d_i^2) = \frac{\pi}{4}(500^2 - 300^2)$$

$$\therefore \boxed{A = 125.66 \times 10^3 \, mm^2}$$

$$k = \sqrt{\frac{2.670 \times 10^9}{125.66 \times 10^3}} = 145.766 \, mm$$

Slenderness ratio $(L/k) = \frac{6000}{145.766} = 41.161$

Sin $L/k < 150$

$$\alpha = \frac{1}{1 - 0.0044 (L/k)}$$

$$k = \frac{d_i}{d_o} = \frac{300}{500}$$

$$\alpha = \frac{1}{1 - 0.0044 \times 41.161}$$

$$k = 0.6$$

$$\alpha = 1.2211$$

Assuming minor shock, $C_m = 1.5 - 2.0 = 1.75$, $C_t = 1 - 1.5 = 1.25$ → [Table 3.1 pg 56]

$$\tau_{max} = \frac{16}{\pi d_o^3}\left[\sqrt{\left(C_m M + \frac{\alpha F d_o (1+k^2)}{8}\right)^2 + (C_t T)^2}\,\right] \times \left(\frac{1}{1-k^4}\right)$$

→ eqn 3.8 (b) pg 51

$$= \frac{16}{\pi (500)^3}\left[\sqrt{\left((1.75)(52.5 \times 10^6) + \frac{(1.2211)(500 \times 10^3)(500)(1+0.6^2)}{8}\right)^2 + (1.25 \times 3.82 \times 10^9)^2}\,\right] \times \frac{1}{(1 - 0.6^4)}$$

$$\cancel{= 4.074\times10^{-8} \times \left[\sqrt{(91.875\times10^{6} + 52.827\times10^{6})^2 + 2.280\times10^{12}} \times 1.201 \times \left[\frac{1}{1-0.6^4}\right]\right]}$$

$$\therefore \tau_{max} = \underline{223.72\ N/mm^2}$$

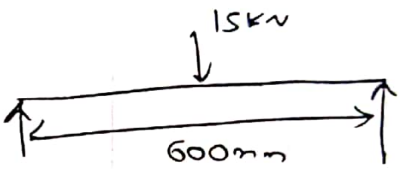
The angular deformation

$$\theta = \frac{584\,TL}{G(d_o^4 - d_i^4)} \rightsquigarrow [\text{eq}^n\ 3.2\ \text{pg}\ 50]$$

$$\theta = \frac{584\,(3.82\times10^{9})\times 6000}{84\times10^{3}(500^4 - 300^4)}$$

$$\boxed{\theta = 2.92^\circ}$$

5) A simply supported shaft has the distance b/w the supports as 600mm the load at the centre is 15kN if the deflection at the centre is to be limited to 0.02mm what should be diameter of the shaft if the shaft diameter is doubled. what will be the deflection at the centre the modulus of elasticity is 210 GPa



soln

$L = 600mm$

$W = 15kN$

$E = 210\,GPa$

$y = 0.02\,mm$

$$y = \frac{WL^3}{48EI} \rightsquigarrow \text{pg } 15$$

$$y = \frac{WL^3}{48\times E\times \frac{\pi}{64}d^4} \rightsquigarrow \text{pg } 13$$

$$d^4 = \frac{WL^3}{48E\,(\pi/64)\times y}$$

$$d^4 = \frac{15\times10^3\times(600)^3\times64}{48\times210\times10^3\times\pi\times0.02}$$

$$\boxed{d = 134.515\ mm}$$

The shaft diameter is doubled

$D = 2\times134.515mm$

~~D = 2 ...6~~

$D = \underline{269.03mm}$

$$y = \frac{WL^3}{48EI} = \frac{15\times10^3\times(600)^3\times6}{48\times210\times10^3\times\pi\times(269.03)}$$

$$\therefore y = \underline{0.00125\ mm}$$

6) A shaft rotating at 300rpm transmits 18KW is supported on bearing at a distance of 3m apart & carries a central load of 2kN selecting C40 steel for shaft ($\sigma_y = 328.6$ MPa) & FOS of 2.5) find suitable diameter of the shaft.

Soln

$n = 300$ rpm

$L = 3000$ mm

$W = 2\times10^3$ N

$\sigma_y = 328.6$ MPa

FOS $= n = 2.5$

$d = ?$

$P = 18$ kN

According to ASME Codes for design

According to max. normal stress theory

$$d_o^3 = \frac{16}{\pi \sigma_{max}}\left[C_m M + \sqrt{(C_m M)^2 + (C_t T)^2}\right]$$

Torque, $T = \frac{9.55\times10^6 P}{n}$ → [eq^n 3.3(a), Pg 50]

$$T = \frac{9.55\times10^6\times18}{300}$$

$$\boxed{T = 573\times10^3 \text{ N-mm}}$$

FOS, $n = \frac{\sigma_y}{\sigma_{max}}$

$$\sigma_{max} = \frac{\sigma_y}{n} = \frac{328.6}{2.5}$$

$$\boxed{\sigma_{max} = 131.44 \text{ N/mm}^2}$$

$$\tau_{max} = \frac{\sigma_{max}}{2} = \frac{131.44}{2}$$

$$\boxed{\tau_{max} = 65.72 \text{ N/mm}^2}$$

$M_{max} = \frac{WL}{4}$ → pg 15

$$= \frac{2\times10^3\times3000}{4}$$

$$\boxed{M_{max} = 1.5\times10^6 \text{ N-mm}}$$

Assuming minor shock

$C_m = 1.5 - 2.0 = 1.75$
$C_t = 1 - 1.5 = 1.25$ → Table 3.1 Pg 56

$$d_o^3 = \frac{16}{\pi \sigma_{max}}\left[C_m M + \sqrt{(C_m M)^2 + (C_t T)^2}\right]$$

$$d_o^3 = \frac{16}{\pi\times(131.44)}\times\left[(1.75\times1.5\times10^6)\times\sqrt{(1.75\times1.5\times10^6)^2 + (1.25\times573\times10^3)^2}\right]$$

$\therefore d = 59.168\,mm$

According to maximum shear stress theory.

$$d_o^3 = \frac{16}{\pi \tau_{max}} \left\{ \left[\sqrt{(C_m M)^2 + (C_t T)^2} \right] \right\}$$

$$= \frac{16}{\pi \times 65.72} \left[\sqrt{(1.75 \times 1.5 \times 10^6)^2 + (1.25 \times 573 \times 10^3)^2} \right]$$

$$\boxed{d = 59.52\,mm}$$

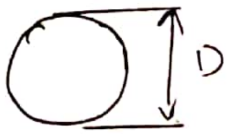
The recommended diameter is 59.52 mm

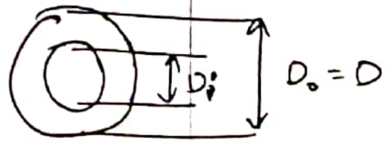
7) Compare the weight strength & stiffness of a hollow shaft with that of solid shaft both shaft have same external diameter length & made of same material the internal diameter of the hollow shaft is 0.6 times the external diameter.

Soln

$D_i = 0.6\,D_o$

$\frac{D_i}{D_o} = 0.6 = k$





For solid shaft

$A = \frac{\pi}{4} D^2$

$J = \frac{\pi}{32} D^4 \rightarrow pg\,(14)$

$R = D/2$

For hollow shaft

$A = \pi/4 \left[D_o^2 - D_i^2 \right]$ $\quad k = \frac{D_i}{D_o}$

$A = \frac{\pi}{4} D_o^2 \left[1 - k^2 \right]$

$A = \pi/4\, D^2 \left[1 - 0.6^2 \right]$ $\quad \therefore D_o = D$

$A = \frac{\pi}{4} \left[0.64 \right] D^2$

$J = \frac{\pi}{32} \left[D_o^4 - D_i^4 \right] \rightarrow pg\,14$

$J = \frac{\pi}{32} D_o^4 \left[1 - k^4 \right]$

$J = \frac{\pi}{32} D_o^4 \left[1 - 0.6^4 \right]$

$J = \frac{\pi}{32} D_o^4 \left[0.8704 \right]$

$R = \frac{D_o}{2} = D/2$

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1) weight (w):-

$w =$ volume $\times$ specific weight

$= (A \times l) \times w$

for hollow shaft, $(W)_H = \frac{\pi}{4} D^2 (0.64) \times l \times w$

For solid shaft, $(W)_S = \frac{\pi}{4} \times D^2 \times l \times w$

on comparing $\frac{W_H}{W_S} = \frac{\cancel{\pi/4}\, \cancel{D^2} (0.64) \times \cancel{l} \times \cancel{w}}{\cancel{\pi/4}\, \cancel{D^2} \times \cancel{l} \times \cancel{w}}$

$\frac{W_H}{W_S} = 0.64$ on comparing hollow shaf is 0.64 times of solid shaft.

2) strength

For shaft strength is based on Torque

$\frac{T}{J} = \frac{\tau}{R}$

$T = \tau (J/R)$

for hollow shaft, $T_H = \tau \left[\frac{\pi/\cancel{32}^{16}\, D^4 (0.8704)}{\cancel{D}/\cancel{2}} \right]$

$T_H = \tau \left[\frac{\pi D^3 (0.8704)}{16} \right]$

For solid shaft $T_S = \tau \left[\frac{\pi/\cancel{32}^{16}\, D^4}{\cancel{D}/\cancel{2}} \right]$

$T_S = \tau \left[\frac{\pi D^3}{16} \right]$

on comparing

$\frac{T_H}{T_S} = \left[\frac{\cancel{\tau} \cancel{\pi} \cancel{D^3} \times 0.8704 \times \cancel{16}}{\cancel{16} \times \cancel{\tau} \cancel{\pi} \cancel{D^3}} \right]$

$\frac{T_H}{T_S} = 0.8704$

on comparing hollow shaft is 0.8704 times of solid shaft.

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3) ~~Sti~~ Stiffness

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\frac{T}{\theta} = \frac{G \times J}{L}$$ → For hollow shaft

$$\left(\frac{T}{\theta}\right)_H = \frac{G}{L}\left(\frac{\pi}{32}(0.8704)D^4\right]$$

for solid shaft

$$\left(\frac{T}{\theta}\right)_S = \frac{G}{L}\left[\frac{\pi}{32}D^4\right]$$

On comparing

$$\frac{(T/\theta)_H}{(T/\theta)_S} = \frac{\cancel{\pi/32}(0.8704)\cancel{D^4}}{\cancel{\pi/32}\cancel{D^4}}$$

$$\frac{(T/\theta)_H}{(T/\theta)_S} = 0.8704$$

On comparing hollow shaft is 0.8704 stiffer than solid shaft.

Shaft mountings

The shaft will usually have mountings of gears, pulleys etc. In such case it is necessary to consider the ~~forces~~ forces induced on the shaft due to this mountings for finding diameter of the shaft.

i) **Forces acting on the shaft due to belt drive**

For belt drives, i) ratio of belt tensions $= \frac{T_1}{T_2} = e^{\mu\theta}$

where μ = friction co-efficient,

T_1 = Tension on tight side

T_2 = Tension on slag side

θ = angle of contact (or) angle of wrap

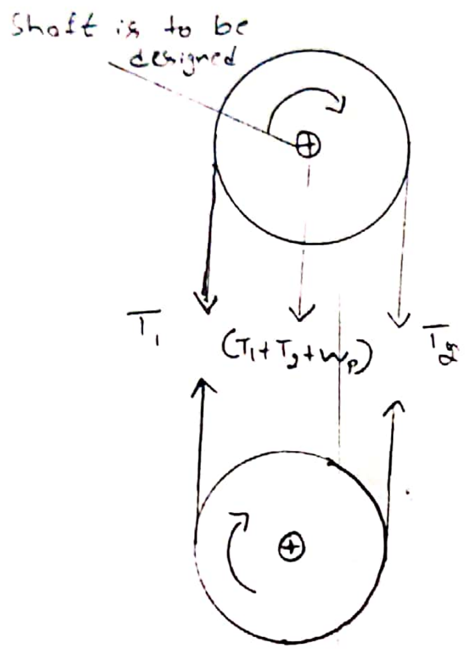
2) maximum Tension, $T_1 = \sigma_t\, bt$

where, b is width of belt, t is thickness of belt

3) Torque, $T = (T_1 - T_2) r_p$

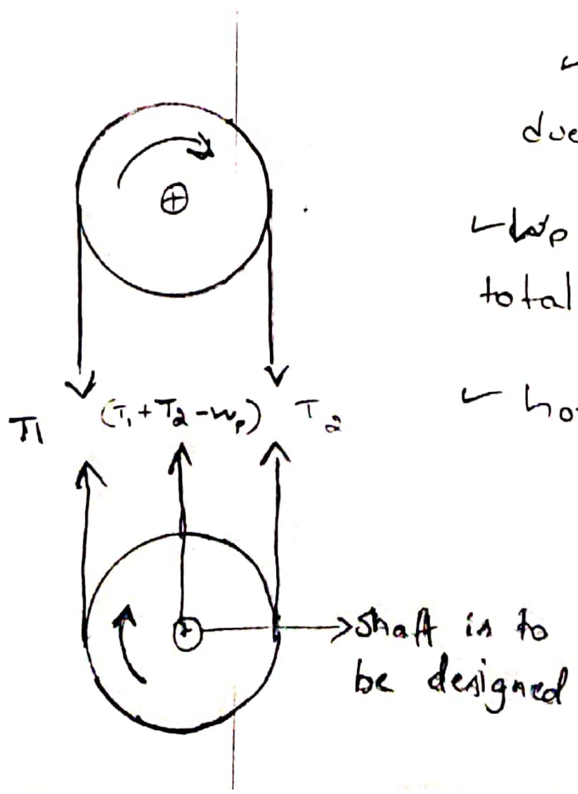
where r_p = radius of pulley

If the shaft recieves (or) transmits power vertically from below (or) downward.



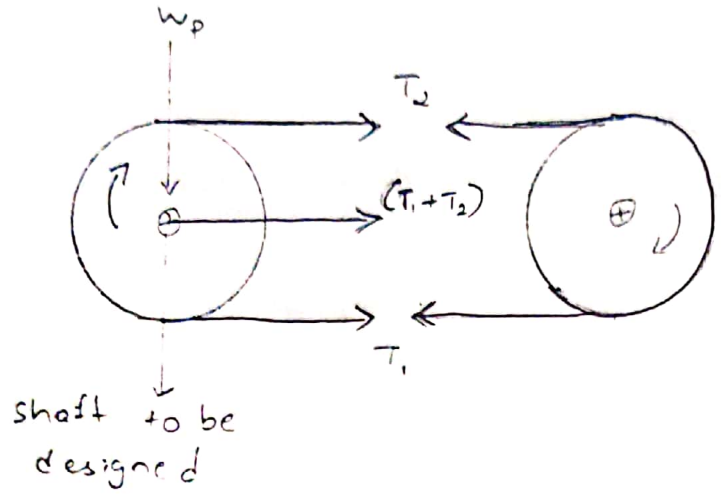
- Down ward forces acting on the shaft due to belt tension $= (T_1+T_2)\downarrow$
- w_p is the weight of the pulley.
- Total downward force $= (T_1+T_2+w_p)\downarrow$
- horizontal forces on the shaft due to tension $= 0$

If the shaft receives (or) Transmits power vertically from above (or) upwards



- upward forces acting on shaft due to belt tension $= (T_1+T_2)\uparrow$
- w_p is the weight of the pulley total Upward force $= (T_1+T_2-w_p)\uparrow$
- horizontal forces on shaft due to tension $= 0$.

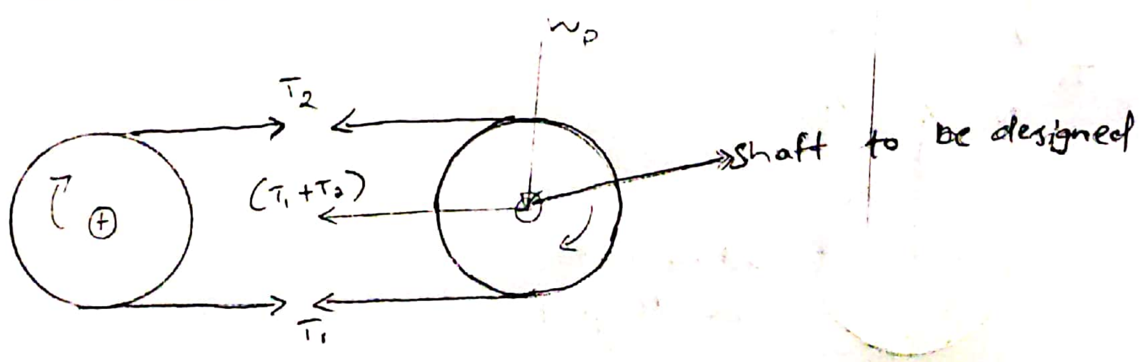
If the shaft recieves (or) Transmits power horizon-tally from the right (or) to the right as shown in figure



- Horizontal forces acting on shaft due to tension $= (T_1 + T_2) \rightarrow$
- W_p is the weight of the pulley.

  The total horizontal force acting on the shaft $= (T_1 + T_2) \rightarrow$
- Total vertical force acting on the shaft $= W_p \downarrow$

If the shaft recieves (or) Transmits power horizontally from the left (or) to the shaft as shown in figure



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- horizontal forces acting on shaft due to tension $= (T_1 + T_2) \leftarrow$

- Total horizontal force acting on the shaft $= (T_1 + T_2) \leftarrow$

- Total vertical force acting on the shaf $= W_p \downarrow$

Problem

1) A line shaft is to transmit 30kW at 160rpm. it is driven by a motor placed directly under it by means of a belt running on a 1m diameter pulley. keyed to the end of the shaft the tension on the tight side of the belt is 2.5 times that on the slag side & the centre of a pulley over hangs 150mm beyond the centre line of the end bearing. determine the diameter of the shaft if allowable shear stress is 60MPa & pulley weights 1600N.

Soln:-

$P = 30kW$

$N = 160 rpm$

$D_p = 1m$

$T_1 = 2.5 T_2$

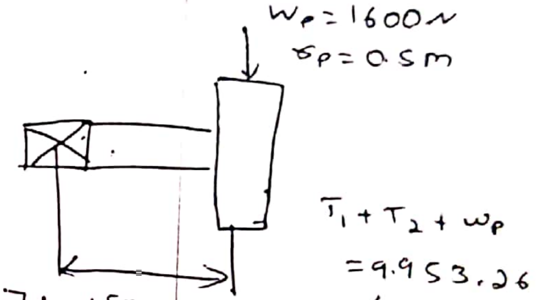
$\tau_{max} = 60 MPa$

$r_p = 0.5$

$D = ?$

According to ASME code.

max. shear stress theory



$$D = \left[\frac{16}{\pi \tau_{max}} \sqrt{(C_m M)^2 + (C_t T)^2}\right]^{1/3}$$

Torque, $T = 9.55 \times 10^6 \times \frac{P}{n}$ ⤳ eqn 3.3(a), 50

$$T = 9.55 \times 10^6 \times \frac{30 \times 10^3}{160}$$

$$\boxed{T = 1.79 \times 10^6 N\text{-}mm}$$

WKT $T = (T_1 - T_2) r_p$

$1.79 \times 10^6 = (2.5T_2 - T_2)(0.5 \times 10^3)$

$\cancel{T_2 = 2.2}$ $\boxed{T_2 = 2386.6N}$

$\therefore T_1 = 2.5 T_2 = 2.5 \times 2386.6$

$\boxed{T_1 = 5966.66N}$

Total vertical force $= T_1 + T_2 + w_p$

$= 5966.66 + 2386.6 + 1600$

$= 9953.26 N (\downarrow)$

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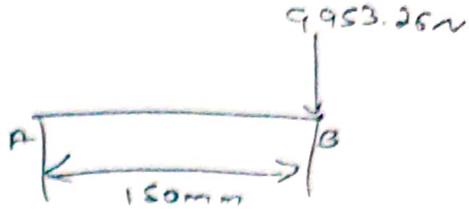
$$D^3 = \frac{16}{\pi \tau_{max}}\left[\sqrt{(C_m M)^2 + (C_t T)^2}\right]$$



moment at point B = 0

$M_A = (9953.26 \times 150)$

$\boxed{M_A = 1.492 \times 10^6 \text{ N-mm}}$

Considering minor shock

$C_m = 1.5 - 2.0 = 1.75$

$C_t = 1.0 - 1.5 = 1.25$ → [Table 3.1 Pg 56]

$$D = \left[\frac{16}{\pi \times (60)}\sqrt{(1.75 \times 1.492 \times 10^6)^2 + (1.25 \times 1.79 \times 10^6)^2}\right]^{1/3}$$

$\boxed{D = 66.28 \text{ mm}}$

2) A section of commercial shaft 2m long b/w bearings carries a 900N pulley at it's mid point. The shaft transmits 21kW at 300rpm & the belt drive is horizontal. the sum of Tension is 6kN. Find the suitable diameter of the shaft & angle of twist b/w the bearings.

Sol^n

$P = 21$ kW

$N = 300$ rpm

$T_1 + T_2 = 6$ kN

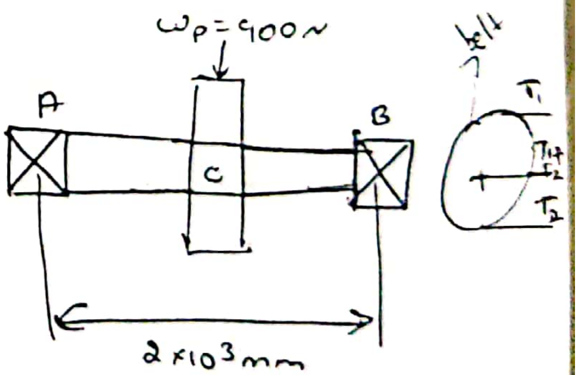
$D = ?$

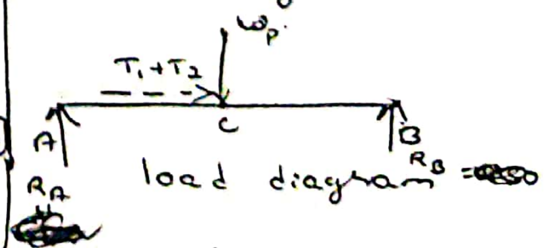
$\theta = ?$

According to ASME code assuming

$\sigma_{max} = 110$ MPa (without keyway)

$\tau_{max} = 55$ MPa (without keyway)

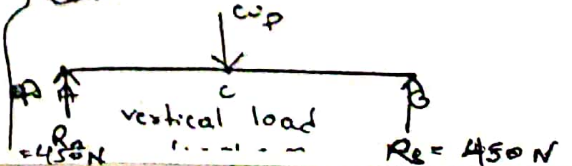




wkT $T = \frac{9.55 \times 10^6 \times P}{n}$ → [eq^n (3.3(a), pg 50)]

$= \frac{9.55 \times 10^6 \times 21}{300}$

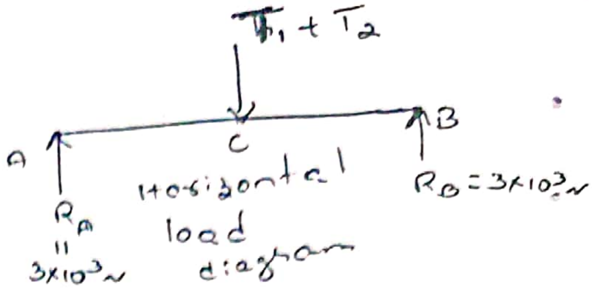
$\boxed{T = 668.5 \times 10^3 \text{ N-mm}}$



According to ASME code

max. normal stress theory

$$D^3 = \frac{16}{\pi \sigma_{max}} \left(C_m M + \sqrt{(C_m M)^2 + (C_t T)^2} \right)$$



max. shear stress theory

$$D^3 = \frac{16}{\pi \tau_{max}} \left(\sqrt{(C_m M)^2 + (C_t T)^2} \right)$$

To find Bending moment (M)

- consider vertical load diagram

since, the load is at the middle the reaction R_A will be equals to $R_B = \frac{900}{2} = 450 N$

since, the beam is a S.S.B carrying load at its centre,

$$M_{max} = \frac{WL}{4} \rightarrow (\text{Pg } 15)$$

$$(M_{max})_{vertical} = \frac{WL}{4} = \frac{900 \times 2\times10^3}{4}$$

$$\therefore \boxed{(M_{max})_V = 450\times10^3 N\text{-mm}}$$

- consider horizontal load diagram

since, the load is at the middle the reaction R_A will be equal to R_B i.e, $R_A = R_B = \frac{T_1+T_2}{2} = \frac{6\times10^3 N}{2}$

$$R_B = R_A = 3\times10^3 N$$

Beam is S.S.B carrying central load

$$\therefore (M_{max})_H = \frac{WL}{4} = \frac{6\times10^3 \times 2\times10^3}{4} = 3\times10^6 N\text{-mm}$$

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The Resultant bending moment,

$$M = \sqrt{(M_b)_V^2 + (M_b)_H^2}$$

$$= \sqrt{(450\times10^3)^2 + (3\times10^6)^2}$$

$$\boxed{M = 3.03\times10^6 \ N\text{-}mm}$$

(we get max. Diameter in shear stress theory so we directly using the shear theory)

max. shear stress theory

$$D^3 = \frac{16}{\pi \tau_{max}}\left(\sqrt{(C_m M)^2 + (C_t T)^2}\right)$$

Take $\tau_{max} = 55 MPa$ (without keyway)

Assume minor shock, $C_m = 1.75$ & $C_t = 1.25$

$$D^3 = \frac{16}{\pi\times55}\times\left(\sqrt{(1.75\times3.03\times10^6)^2 + (1.25\times \cancel{6}\,668.5\times10^3)^2}\right)$$

$\boxed{\cancel{D = 78.89\,mm}}$

$\therefore$ $\boxed{D = 79.21mm}$ $\approx$ 80mm

(approximate the value from the Table 3.5(a) Pg 57)

ii) Angle of twist (θ)

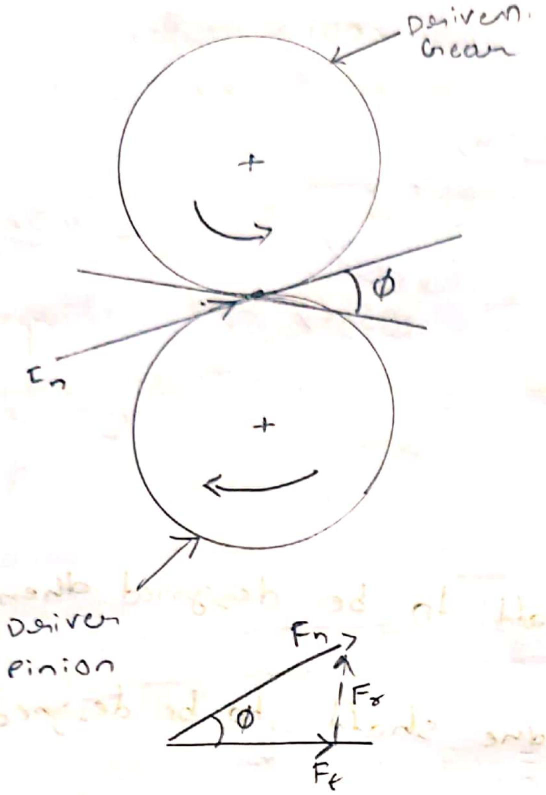
$$\theta = \frac{584TL}{Gd^4}$$ → eqn 3.2 Pg 50

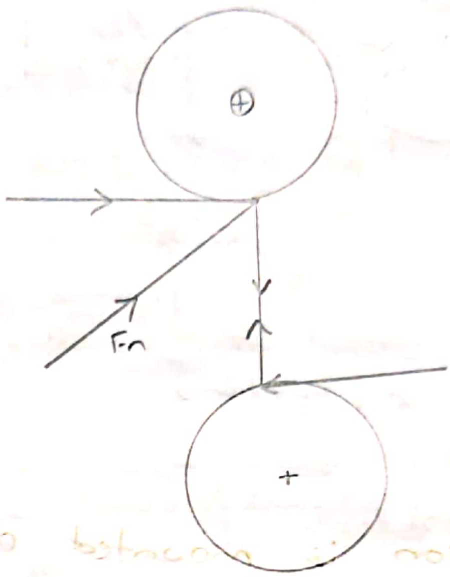
Take, $G = 80GPa = 80\times10^3 \ N/mm^2$

$$\theta = \frac{584\times668.5\times10^3\times2\times10^3}{80\times10^3\times(80^4)}$$

$\therefore \theta = 0.238°$

Forces acting on shaft due to Gear drive

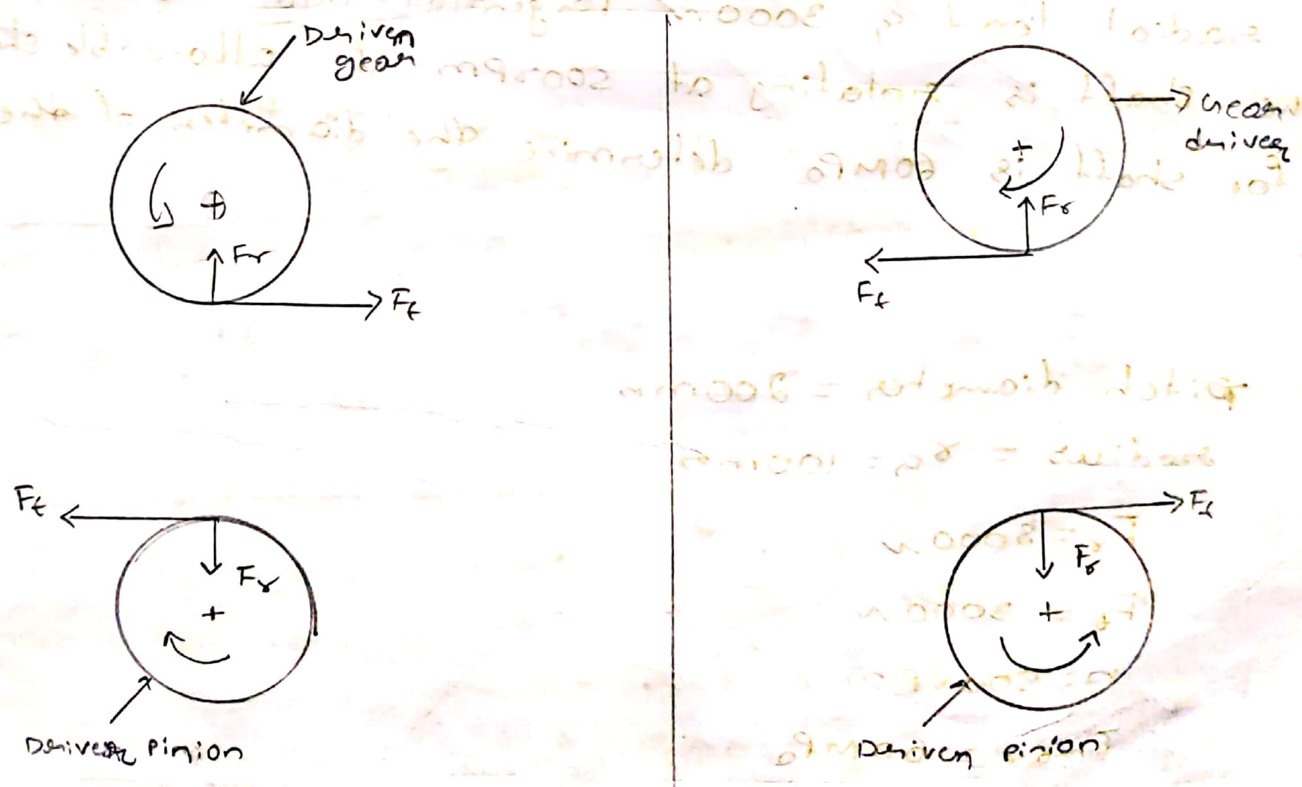




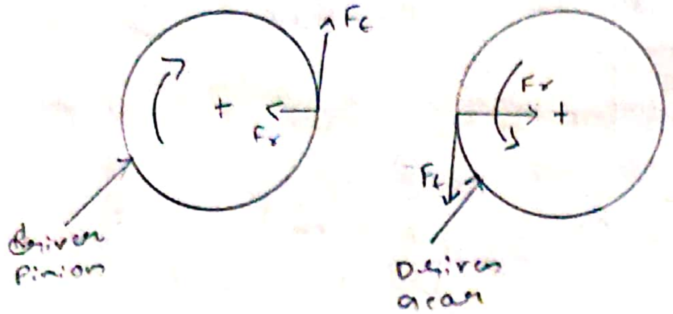
$T = F_t \times r_g$

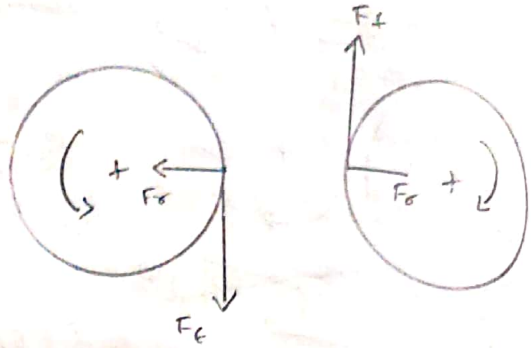
$F_r = F_t \times \tan\phi$

1) Gears are placed one above the other (or) one below the other



2) Gears are placed side by side





Torque, $T = F_t \times r_g$

$F_r = F_t \times \tan\phi$

✓ If pinion is mounted on the shaft to be designed then consider pinion forces.

✓ ~~If the~~ the Gear is mounted on the shaft to be designed then consider gear forces.

Problem

1) A shaft is mounted on bearing @400mm apart & carrier at its midle. A gear of 200mm pitch diameter. The Gear carries 8000N radial load & 3000N tangential load as shown in figure the shaft is rotating at 500rpm if allowable shear stress for shaft is 60MPa determine the diameter of the shaft.

Solⁿ:-

Ggn= Pitch diameter = 200mm

radius = r_g = 100mm

$F_r = 8000N$

$F_t = 3000N$

$n = 500rpm$

$\tau_{max} = 60MPa$

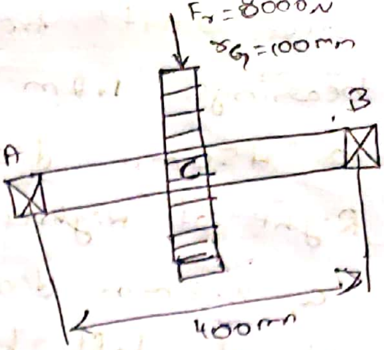
$d = ?$

i) Torque

$T = F_t \times r_G$

$= 3000 \times 100$

$\therefore T = 3 \times 10^5 \ N\text{-}mm$

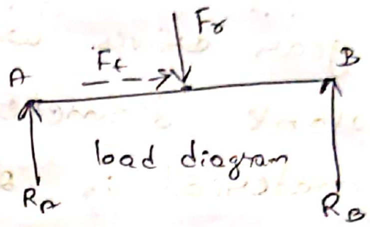


ii) Bending moment (M)

consider vertical load, → pg (5)

$$(M_b)_V = \frac{WL}{4} = \frac{8000 \times 400}{4}$$

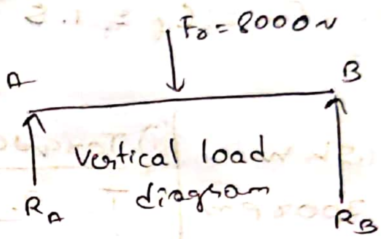
$\therefore (M_b)_V = 800 \times 10^3 \ N\text{-}mm$

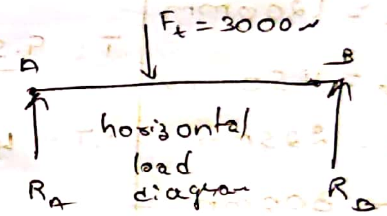


consider horizontal load,

$$(M_b)_H = \frac{WL}{4} = \frac{(3000) \times 400}{4}$$

$\therefore (M_b)_H = 300 \times 10^3 \ N\text{-}mm$





~~The~~ resultant bending moment

$$M = \sqrt{(M_b)_V^2 + (M_b)_H^2}$$

$$= \sqrt{(800 \times 10^3)^2 + (300 \times 10^3)^2}$$

$$\therefore \boxed{M = 854.400 \times 10^3 \ N\text{-}mm}$$

~~as~~ according to max. shear stress

$$d^3 = \frac{16}{\pi \tau_{max}} \left[\sqrt{(C_m M)^2 + (C_t T)^2} \right]$$

assuming minor shock

$C_m = 1.5 - 2.0 \rightarrow 1.75$

$C_t = 1.0 - 1.5 \rightarrow 1.25$

$$d^3 = \frac{16}{\pi \times 60} \left[\sqrt{(1.75 \times 854.4 \times 10^3)^2 + (1.25 \times 3 \times 10^5)^2} \right]$$

$$\boxed{d = 50.767 mm}$$

standard shaft sizes in mm
Table 3.5 (a) pg 57

$$\boxed{d = 56 \ mm}$$

2) A mild steel shaft transmit 15kW at 300rpm. it is supported on two bearings 1.2m apart. The shaft receives power through a 450mm diameter pulley mounted at 300mm to the Right of Right bearing. The power is given out through a 300mm diameter gear mounted at 250mm to the right of left bearing. The belt drive is Horizontal & the gear drives with a downward tangentially force. find suitable diameter of the shaft if yield stress for shaft material is 234MPa & factor of safety is 2. Take $C_m = C_t = 1.5$. Tension ratio of belt is 3.

<u>Solⁿ</u>

$P = 15kW$

$n = 300rpm$

$l = 1200mm$

$d_p = 450mm$

$r_p = 225mm$

$d_G = 300mm$

$r_G = 150mm$

$D = ?$

$\sigma_y = 234MPa$

$n = 2$

$C_m = 1.5$

$C_t = 1.5$

$\frac{T_1}{T_2} = 3$

i) <u>Torque</u>

$T = \frac{9.55 \times 10^6 (P)}{n}$ → eqⁿ 3.3(a) PJ 50

$T = \frac{9.55 \times 10^6 \times 15}{300}$

$\therefore T = 477.5 \times 10^3$ N-mm

~~ii) Bending moment (M)~~

~~Consider vertical load diagram~~

$\cancel{(M)_v = \frac{WL}{4}} = \frac{F_t L}{4}$

Consider Gear at point (D)

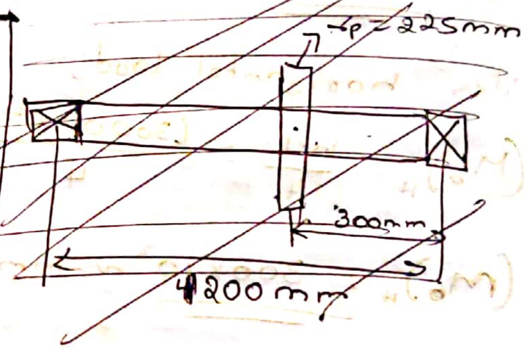
WKT, $T = F_t \times r_G$

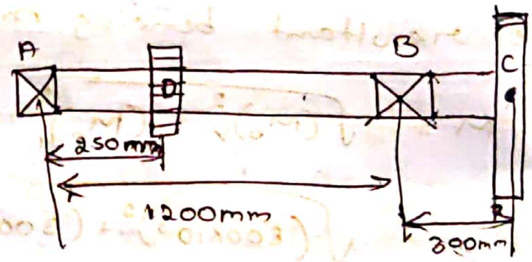
$F_t = \frac{T}{r_G} = \frac{477.5 \times 10^3}{150}$

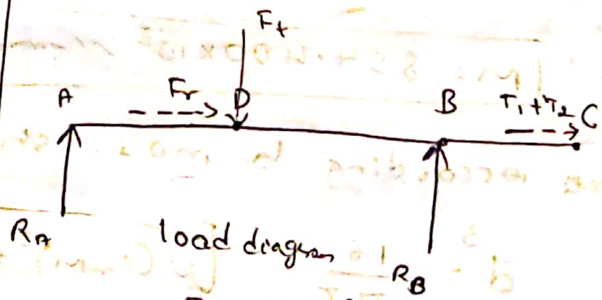
$\therefore \boxed{F_t = 3.183 \times 10^3 N}$ ↓

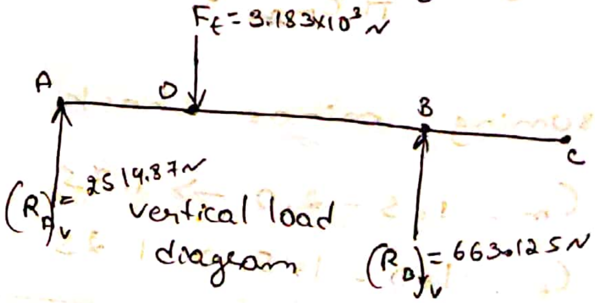
$\cancel{(M_D)_v = \frac{3.183 \times 10^3 \times 1200}{4}}$

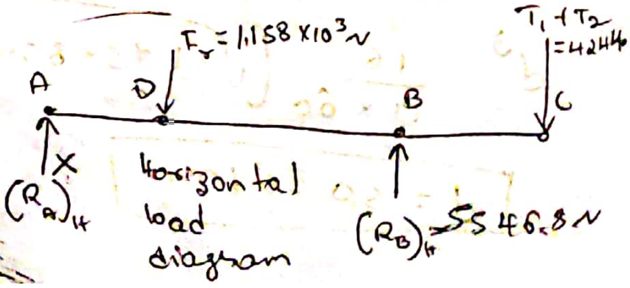
$\cancel{(M_D)_v = 954.9 \times 10^3 \text{ N-mm}}$











~~Consider load horizontal load diagram~~

consider Gear at point 'D'

Radial force, $F_r = F_t \times \tan\phi = 3.183\times10^3 \times \tan 20$

$\boxed{F_r = 1.1585\times10^3}$ $\rightarrow$

Consider pulley at point 'C'

WKT $T = (T_1 - T_2) r_p$

Given $\frac{T_1}{T_2} = 3$ $T_1 = 3T_2$

$T = (3T_2 - T_2) \times 225$

$\therefore T = 2T_2 \times 225$

$T_2 = \frac{477.5\times10^3}{2\times225}$

$\boxed{T_2 = 1061.11 \text{ N-mm}}$

$\frac{T_1}{T_2} = 3$

$T_1 = 3T_2$

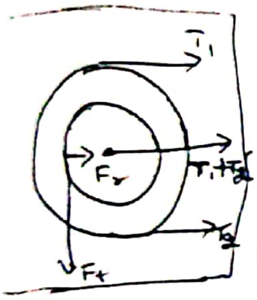
$T_1 = 3\times1061.11$

$T_1 = \underline{3183.33 \text{ N-mm}}$

The total tension $= T_1 + T_2$

$= T_1 + T_2 = 1061.11 + 3183.33$

$\therefore \boxed{T_1 + T_2 = 4244.44 \text{ N-mm}}$ $(\rightarrow)$



Consider vertical load diagram

$(R_A)_v + (R_B)_v = 3.183\times10^3$

Taking moment about point A

$(F_t \times 250) - (R_{Bv} \times 1200) = 0$

$(3.183\times10^3 \times 250) - (R_{Bv} \times 1200) = 0$

$(R_B)_v = \frac{3.183\times10^3\times250}{1200}$

$\boxed{(R_B)_v = 663.125 \text{ N}}$

$(R_A)_v + (R_B)_v = 3.183\times10^3$

$(R_A)_v = 3.183\times10^3 - 663.125$

$\therefore \boxed{(R_A)_v = 2519.875 \text{ N}}$

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Bending moment at points.

$M_{bvA} = 0$

$M_{bvB} = 0$

$M_{bvC} = 0$

$M_{bvD} = (R_{Av} \times 250) = (2519.875 \times 250)$

$M_{bvD} = 629968.75$ N-mm

$\therefore \boxed{M_{bvD} = 629.96 \times 10^3 \text{ N-mm}}$

Consider Horizontal load diagram

$(R_A)_H + (R_B)_H = F_r + (T_1 + T_2)$

$(R_A)_H + (R_B)_H = 1.158 \times 10^3 + 4244.44$

$(R_A)_H + (R_B)_H = 5.40244 \times 10^3$

Taking moment about point A,

$(1158 \times 250) - R_{BH} \times 1200 + (4244.44 \times 1500) = 0$

$\boxed{R_{BH} = 5546.8 \text{ N}}$

$(R_A)_H = 5.4024 \times 10^3 - 5546.8$

$\therefore \boxed{(R_A)_H = -144.4 \text{ N}}$

Bending moment at the points

$\therefore M_{bAH} = 0$

$M_{bDH} = (-5546.8 \times 950) + (4244.44 \times 1250)$

$\therefore \boxed{M_{bDH} = 36090 \text{ N-mm}} = 36.09 \times 10^3 \text{ N-mm}$

Scanned with CamScanner

$M_{bBH} = 4244.44 \times 300$

$\therefore \boxed{M_{bBH} = 1.273 \times 10^6 \text{ N-mm}}$

$\therefore M_{bDH} = 0$

Resultant Bending moment (M).

$\cancel{M_A = \sqrt{(M_{bAV})^2 + (M_{bBH})^2 + (M_{bCV})^2 + (M_{bDV})^2}}$

$M_A = \sqrt{(M_{bAV})^2 + (M_{bAH})^2} = 0$

$M_B = \sqrt{(M_{bBV})^2 + (M_{bBH})^2} = \sqrt{0 + (1.273 \times 10^6)^2}$

$\therefore \boxed{M_B = 1.273 \times 10^6 \text{ N-mm}}$

$M_C = \sqrt{(M_{bCV})^2 + (M_{bCH})^2} = \cancel{\sqrt{0 + (36.09 \times 10^3)^2}}$

$\therefore \cancel{\boxed{M_C = 36.09 \times 10^3 \text{ N-mm}}}$ $\therefore \boxed{M_C = 0}$

$\cancel{M_D = \sqrt{(M_{bDV})^2 + (M_{bDH})^2} = \sqrt{(629.96 \times 10^3)^2 + 0}}$

$\therefore \cancel{\boxed{M_D = 629.96 \times 10^3 \text{ N-mm}}}$

$M_D = \sqrt{(M_{bDV})^2 + (M_{bDH})^2} = 631.06 \times 10^3 \text{ N-mm}$

The maximum bending moment occurs at point B

$\therefore \boxed{M_{max} = 1.273 \times 10^6 \text{ N-mm}}$

$n = \frac{\sigma_y}{\sigma_{max}}$

$\sigma_{max} = \frac{234}{2} = 117 \text{ MPa}$

$\tau_{max} = \frac{\sigma_{max}}{2} = \frac{117 \text{ MPa}}{2}$

$\boxed{\tau_{max} = 58.5 \text{ MPa}}$

ASME,
According to ~~the~~ max. shear stress theory.

$$D^3 = \frac{16}{\pi \tau_{max}} \left(\sqrt{(C_m M)^2 + (C_t T)^2}\right)$$

~~Assume~~
~~Sudden shock~~
$C_m = 1.5$
$C_t = 1.5$.

$$= \frac{16}{\pi \times 58.5} \left[\sqrt{(1.5 \times 1.273 \times 10^6)^2 + (1.5 \times 477.5 \times 10^3)^2}\right]$$

$$\therefore \boxed{D = 56.204\ mm}$$

From the data hand book
the standard shaft sizes in mm [Table 3.5 Pg 57]

$$\therefore \boxed{D = 63\ mm}$$

3) A shaft supported by two bearing placed 1m apart. If 600mm diameter pulley is mounted at ~~the~~ a distance of 300mm to right of left bearing. & this drives a pulley directly below it.. with a help of belt having maximum tension of 2.25 kN. Another pully 400mm diameter is placed 200mm to the left of Right bearing & his driven with the help of electric motor & belt which is horizontal to the rigt, the angle of contact for ~~the~~ both Pulley is 180° & $\mu = 0.24$. Determine the suitable diameter for a solid shaft allowing working stress of 63 MPa in tension & 42 MPa is shear for the material of the shaft. assume the torque of one pulley is equal to that on the other pulley.

Sol^n

$T_1 = 2.25\,kN$

$\theta = 180^\circ$

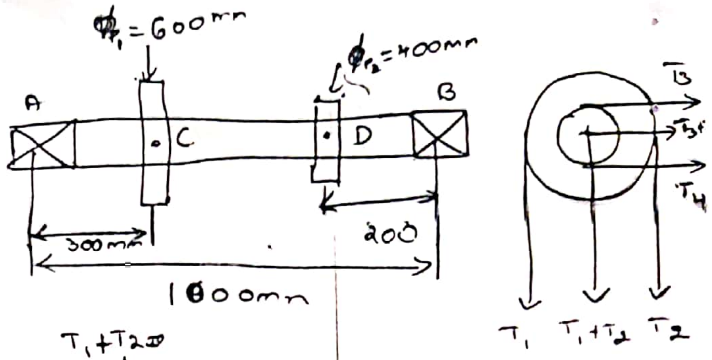
$\mu = 0.24$

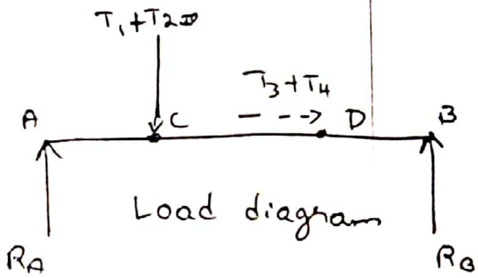
$D = ?$

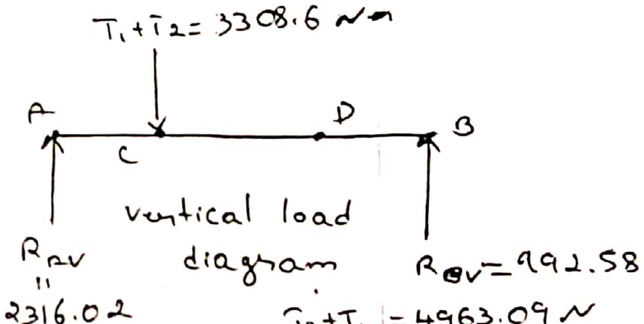
$\sigma_{max} = 63\,MPa$

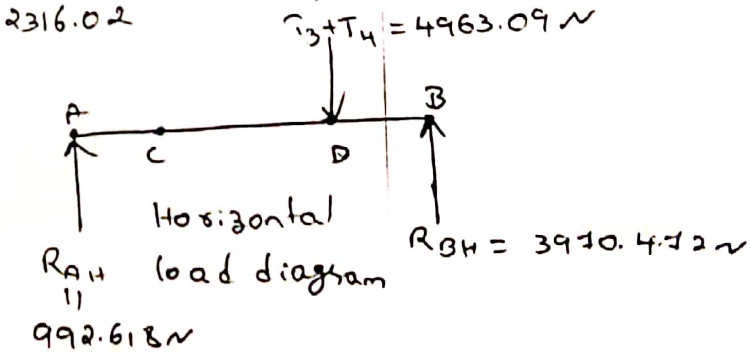
$\tau_{max} = 42\,MPa$

$$\boxed{\frac{T_1}{T_2} = \frac{T_3}{T_4} = T}$$









i) To find Torque.

$T = (T_1 - T_2) r_p$

Ratio of belt tensions,

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{T_1}{T_2} = e^{(0.24 \times 180 \times \pi/180)}$$

$$T_2 = \frac{T_1}{e^{(0.24\times\pi)}} = \frac{2.25\times10^3}{e^{(0.24\times\pi)}}$$

$$\boxed{T_2 = 1058.6\,N}$$

$$T = (T_1 - T_2) r_p = (2.25\times10^3 - 1058.6)\times 300$$

$$\boxed{T = 357.419\times10^3\,N\text{-}mm}$$

Consider pulley at point D'

WKT, Torque, $T = (T_3 - T_4) r_p$

$\longrightarrow$ Given $\frac{T_1}{T_2} = \frac{T_3}{T_4} = \frac{2.25\times10^3}{1058.6}$

$$357.419\times10^3 = T_4\left(\frac{T_3}{T_4} - 1\right) r_p$$

$$357.419\times10^3 = T_4(2.1254 - 1)\times 200$$

$$\boxed{T_4 = 1587.96\,N}$$

$\cancel{T_3 = 2.}$ $\frac{T_3}{T_4} = \frac{T_1}{T_2} = \frac{2.25\times10^3}{1058.6}$

$T_3 = \frac{2.25\times10^3}{1058.6} \times 1587.9\cancel{610}$

$\boxed{\cancel{T_3 = 3336.74 N}}$

$\therefore \boxed{T_3 = 3375.1366 N}$

consider vertical load diagram

$\cancel{M_{bAV} = 0}$

$\cancel{M_{bCV} = 0}$, 1

$R_{AV} + R_{BV} = T_1 + T_2$

$R_{AV} + R_{BV} = 3308.6 N$

Taking moment about point A

$(3308.6 \times 300) - R_{BV} \times 1000 = 0$

$\boxed{R_{BV} = 992.58 N}$

$R_{AV} + R_{BV} = 3308.6 N$

$R_{AV} = 3308.6 - R_{BV}$

$R_{AV} = 3308.6 - 9992.58$

$\boxed{R_{AV} = 2316.02 N}$

Bending moment at points

$M_{bAV} = 0$

$M_{bCV} = (R_{AV} \times 300) = 2316.02 \times 300$

$\boxed{M_{bCV} = 694.806 \times 10^3 N\text{-}mm}$

$M_{bDV} = (R_{AV} \times 800) - (3308.6 \times 500) = (2316.02 \times 800) - (3308.6 \times 500)$

$\therefore \boxed{M_{bDV} = 198.516 \times 10^3 N\text{-}mm}$

$M_{bBV} = 0$

consider Horizontal load diagram

$R_{AH} + R_{BH} = T_3 + T_4$

$R_{AH} + R_{BH} = 4963.09\,N$

Taking moment about point A

$(4963.09 \times 800) - (R_{BH} \times 1000) = 0$

$\boxed{R_{BH} = 3970.472\,N}$

$R_{AH} = 4963.09 - R_{BH} = 4963.09 - 3970.472$

$\boxed{R_{AH} = 992.618\,N}$

Taking Bending moment at the points

$M_{bAH} = 0$

$M_{bCH} = R_{AH} \times 300 = 992.618 \times 300$

$\boxed{M_{bCH} = 297.78 \times 10^3\,N\text{-}mm}$

$M_{bDH} = R_{AH} \times 800 = 992.618 \times 800$

$\therefore \boxed{M_{bDH} = 794.0944 \times 10^3\,N\text{-}mm}$

$\boxed{M_{bBH} = 0}$

Resultant Bending moment (M)

$M_A = \sqrt{(M_{bAV})^2 + (M_{bAH})^2} = 0$

$M_B = \sqrt{(M_{bBV})^2 + (M_{bBH})^2} = 0$

$M_C = \sqrt{(M_{bCV})^2 + (M_{bCH})^2} = \sqrt{(694.806 \times 10^3)^2 + (297.78 \times 10^3)^2}$

$\boxed{M_C = 755.9287 \times 10^3\,N\text{-}mm}$

$$M_D = \sqrt{(M_{bDM})^2 + (M_{bDH})^2} = \sqrt{(198.516\times10^3)^2 + (794.0944\times10^3)^2}$$

$$\boxed{M_D = 818.5319\times10^3 \ N\text{-}mm}$$

Note: The max. Bending moment occurs at point D

$$\boxed{M_{max} = 818.5319\times10^3 \ N\text{-}m}$$

max. normal stress theory:

$$D^3 = \frac{16}{\pi \sigma_{max}}\left[(C_m M) + \sqrt{(C_m M)^2 + (C_t T)^2}\right]$$

assume minor shock

$C_m = 1.5$

$C_t = 1.5$

$$D^3 = \frac{16}{\pi\times63}\left[(1.5\times818.5319\times10^3) + \sqrt{(1.5\times818.5319\times10^3)^2 + (1.5\times357.419\times10^3)^2}\right]$$

$$\boxed{D = 59.208 \ mm}$$

max shear stress theory.

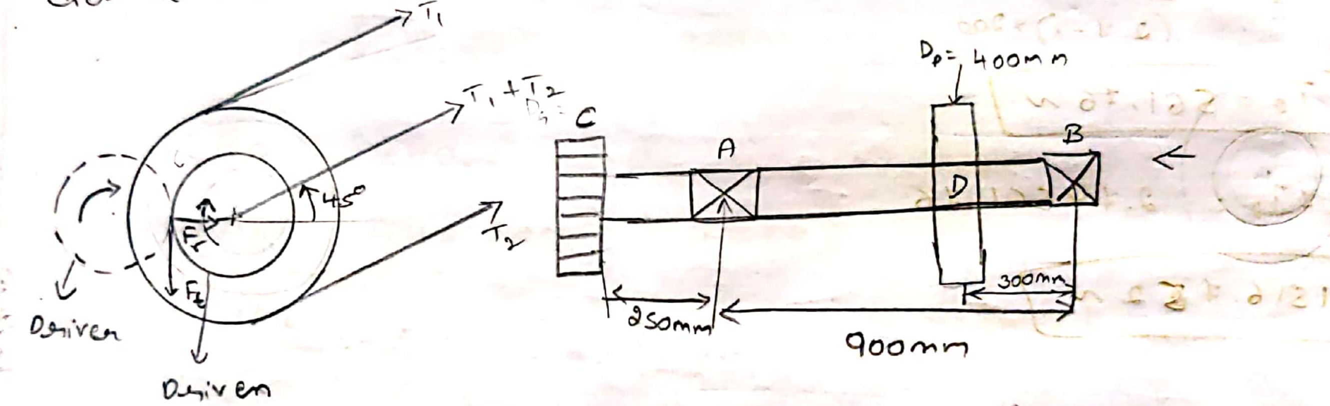
$$D^3 = \frac{16}{\pi \tau_{max}}\left(\sqrt{(C_m M)^2 + (C_t T)^2}\right)$$

$$= \frac{16}{\pi\times42}\left(\sqrt{(1.5\times818.5319\times10^3)^2 + (1.5\times357.419\times10^3)^2}\right)$$

$$\boxed{D = 54.5650mm}$$

standard the size of shaft Using table.

4) A ~~steel~~ steel shaft of 0.9m b/w bearing recieves 18kW at 900rpm through 20° involute gear of 2mm module & 100 teeth located at 250mm to the left of left bearing & it is driven by the gear placed directly behind it. The power is transmitted by a 400mm diameter polley to another polley placed towards front & above it at ~~an~~ an angle of 45° to the horizontal. The polley is located at a distance of 300mm to the left of right bearing the ~~tensile~~ tension ratio is 2.7. Design a hollow shaft taking allowable shear stress has 72MPa. take diameter ratio as 0.5.



$P = 18kW$

$n = 900rpm$

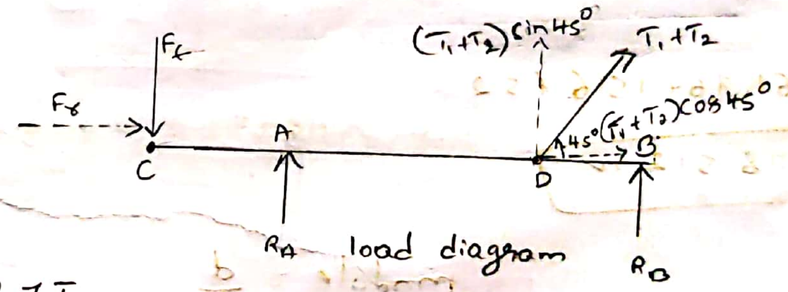
$\frac{T_1}{T_2} = 2.7 \Rightarrow T_1 = 2.7T_2$

$\frac{d_i}{d_o} = 0.5$

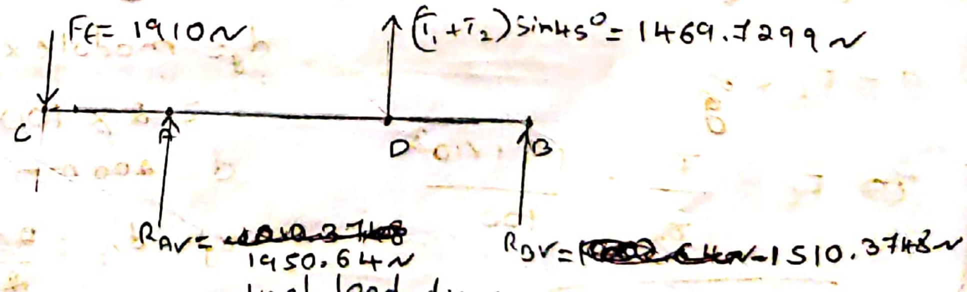
$T = (T_1 - T_2) r_p$

$\frac{P}{T}$

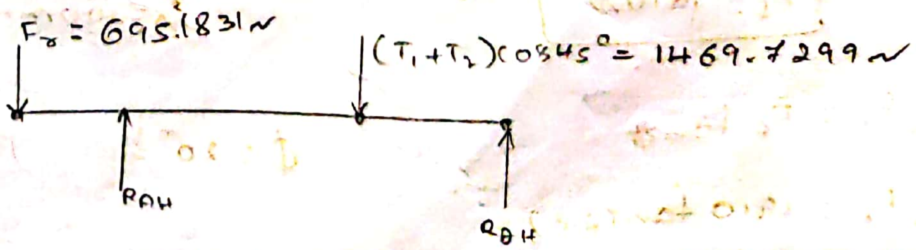
$k = 0.5$



load diagram



vertical load diagram



Horizontal load diagram

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$$T = \frac{9.55\times10^6 (P)}{n} \rightsquigarrow [\text{eq}^n\ 3.3(a),\ Pg\ 50]$$

$$T = \frac{9.55\times10^6 \times 18}{900}$$

$$\boxed{T = 191\times10^3\ N\text{-}mm}$$

$$T = (T_1 - T_2)\, r_p$$

$$191\times10^3 = (2.7T_2 - T_2)\,200$$

$$191\times10^3 = T_2\,(2.7-1)\,200$$

$$T_2 = \frac{191\times10^3}{(2.7-1)\times200}$$

$$\boxed{T_2 = 561.76\ N}$$

$$T_1 = 2.7T_2 = 2.7\times561.76$$

$$\boxed{T_1 = 1516.752\ N}$$

$$T_1 + T_2 = 561.76 + 1516.752$$

$$\boxed{T_1+T_2 = 2078.512\ N}$$

To find F_t

$$\text{module} = \frac{d}{z}$$

$$T = F_t \times r_g$$

$$d_g = \text{module}\times z$$

$$= 2\times100$$

$$d = 200\ mm$$

$$F_t = \frac{T}{r_g} = \frac{191\times10^3}{100}$$

$$r_g = d/2 = \frac{200}{2}$$

$$\therefore \boxed{F_t = 1910N}$$

$$r_g = 100mm$$

$$F_r = F_t \tan\phi$$

$$\phi = 20^\circ$$

$$F_r = 1910\tan(20^\circ)$$

$$\boxed{F_r = 695.1831N}$$

Scanned with CamScanner

Consider vertical load diagram

$(T_1 + T_2) \sin 45° = 1469.7299 N$

$R_{Av} + R_{Bv} = \cancel{1469}.1910 - 1469.7299$

$R_{Av} + R_{Bv} = 440.2701 N$

Take moment at A

$-1469.7299 \times 600 - R_{Bv} \times 900 = 1910 \times 250$

$\frac{-1469.7299 \times 600 - 1910 \times 250}{900} = R_{Bv}$

$\boxed{R_{Bv} = -1510.3748 N}$

$R_{Av} = 440.270 - R_{Bv}$

$R_{Av} = 440.270 + 1510.3748$

$\boxed{R_{Av} = 1950.64 N}$

moment at all points

$M_{Av} = 1910 \times 250 = 477500 N\text{-}mm$

$M_{cv} = 0$

~~M 1950.64 x 300 = 5C1469.~~

$M_{Dv} = \cancel{1950.64 \times 300}\ 1510.3748 \times 300 = 453112.44 N$

$M_{Bv} = 0$

consider horizontal load diagram

$R_{A_H} + R_{B_H} = 695.1831 + 1469.7299$

$\boxed{R_{A_H} + R_{B_H} = 2164.913 N}$

moment about A

$1469.7299 \times 600 - R_{BH} \times 900 - 695.1831 \times 250 = 0$

$R_{BH} = 786.7135 N$

$\boxed{R_{BH} = 1378.19 N}$

VB

$M_A = 0$

$M_B = 695.183 \times 250 = 173.79 \times 10^3$

$M_C = -786.713 \times 300 = -236.013 \times 10^3$

$M_D = 0$

resultand BM

$M_A = 0$ $M_B = 508.1429 \times 10^3$ N-mm

$M_D = 0$ $M_C = 510.87 \times 10^3$ N-mm

$$d_o^3 = \frac{16}{\pi \tau_{max}} \left\{ \sqrt{(C_m M)^2 + (C_t T)^2} \right\} \left(\frac{1}{1-k^4} \right)$$

$$= \frac{16}{\pi \times 72} \left\{ \sqrt{(1.75 \times 510.892 \times 10^3)^2 + (1.25 \times 191 \times 10^3)^2} \right\} \left(\frac{1}{1-(0.5)^4} \right)$$

$d_o = \underline{41.17 mm}$

stand $d_o = 41.17 \simeq 45 mm$

$d_i = \underline{22.5 mm}$

$d_o = 41.7 mm$

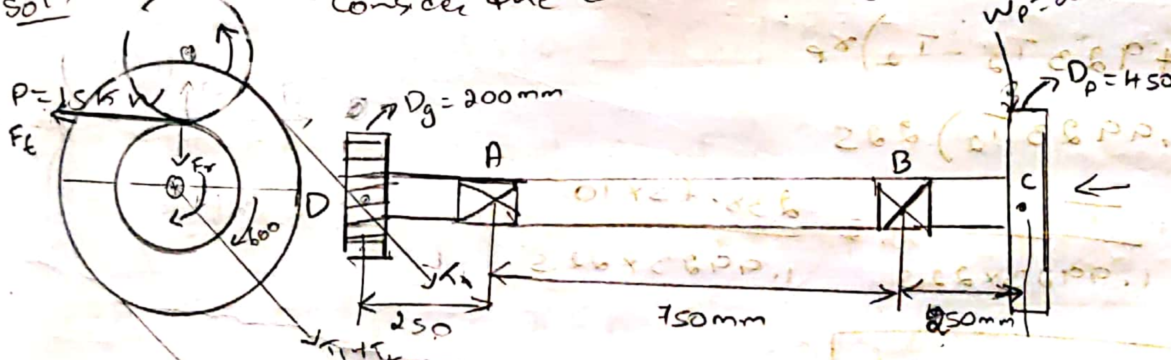
$d_i = 20.58 mm$

5) A machine shaft Running at 600 rpm is supported on bearings 750mm apart 15kW of power is supplied to the shaft through a 450mm pulley located at 250mm to the Right of Righ bearing. The power is transmitted from shaft through a 200mm gear located at 250mm to the left of left bearing. The belt is at an angle 60° downward with horizontal. The pulley of 800N to provide some fly wheel effect the angle of contact of belt is 157° & co-efficient of friction b/w belt & pulley is 0.4. The gear is in mesh with another gear located directly above the Shaft if the shaft material selected has an ultimate tensile strength of 500MPa & yield stress of 310MPa. Determine the necessary diameter of the shaft.

angle of contact of belt = angle of wrap of belt = angle of wrap

↳ consider the effect of key way, ~~exact~~

Soln



n = 600 rpm

P = 15kW

d_P = 450mm

d_G = 200mm

W_P = 800N

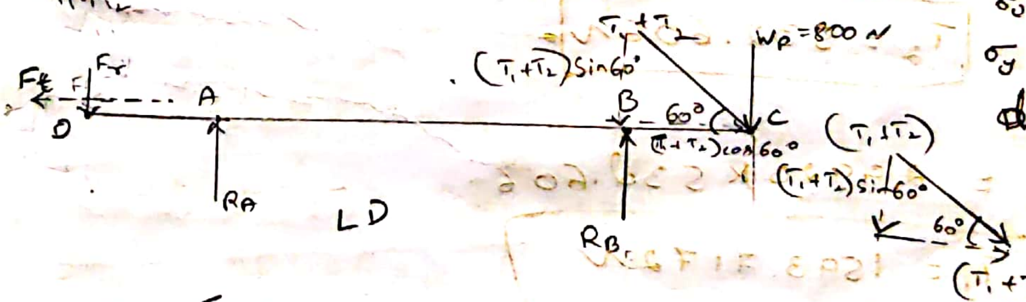
θ = 157°

μ = 0.4

σ_u = 500MPa

σ_y = 310MPa

d = ?

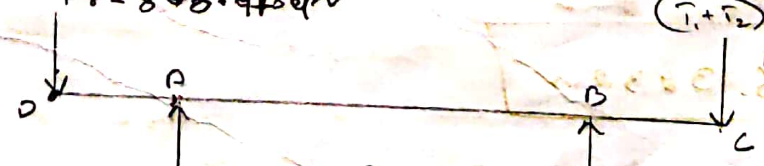


LD

$(T_1+T_2)\cos 60°$

$(T_1+T_2)\sin 60° + W_P$ = 2641.4499 N

1841.4 + 800 =



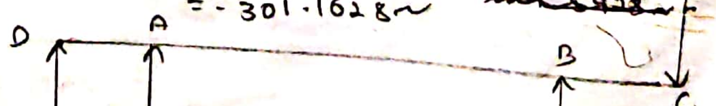
VLD

F_r = 868.9789 N

R_{Av} = -301.1628 N

R_{Bv} = 3811.5916 N

$(T_1+T_2)\cos 60°$ = 1063.1616 N



HLD

F_t = 2387.5 N

R_{AH} = -1946.053 N

R_{BH} = 621.71540 N

To find Torque.

$$T = \frac{9.55\times10^6 (P)}{n} \rightsquigarrow [3.3(a) \text{ Pg } 50]$$

$$T = \frac{9.55\times10^6\times15}{600} = \underline{238.75\times10^3 \text{ N-mm}}$$

Consider pulley at point 'C'

WKT Torque, ~~[illegible]~~ $T = (T_1 - T_2) r_p$

WKT $\frac{T_1}{T_2} = e^{\mu\theta} = e^{(0.4)\times157^\circ \times \pi/180}$

$$\boxed{\frac{T_1}{T_2} = 2.9923}$$

$r_p = \frac{D_p}{2} = \frac{450}{2}$

$r_p = 225$

$T_1 = 2.9923 T_2$

$T = (2.9923 T_2 - T_2) r_p$

$T = (1.9923 T_2) 225$

$$T_2 = \frac{T}{1.9923\times225} = \frac{238.75\times10^3}{1.9923\times225}$$

$$\boxed{T_2 = 532.606 \text{ N}}$$

$T_1 = 2.9923 \times 532.606$

$$\boxed{T_1 = 1593.7172 \text{ N}}$$

$$\therefore \boxed{T_1 + T_2 = 2126.3232 \text{ N}}$$

$(T_1 + T_2)\cos 60^\circ = \underline{1063.1616 \text{ N}}$

$(T_1 + T_2)\sin 60^\circ = \underline{1841.4499 \text{ N}}$

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onsider the gear at D

$r_G = \frac{200}{2}$

$r_G = 100$

Torque, $T = F_t \times r_G$

$F_t = \frac{T}{r_G} = \frac{238.75\times10^3}{100}$

$\boxed{F_t = 2387.50\ N}$

$F_r = F_t \times \tan\phi$

we assume the $\phi = 20^\circ$

$F_r = F_t \times \tan(20^\circ)$

$= 2387.5 \times \tan 20$

$\therefore \boxed{F_r = 868.9789\ N}$

Consider ~~horizontal~~ to vertical load diagram

From diagram

$R_{Av} + R_{Bv} = 868.9789 + 2641.4499$

$\boxed{R_{Av} + R_{Bv} = 3510.4288\ N}$

Take the moment about point A

~~$-868.9789 \times 250 + R_{Bv} \times 750 + 2641.4499 \times 1000 = 0$~~

~~$\therefore R_{Bv} = 3811.5928\ N$~~ ~~$\therefore R_{Bv} = 3232.2735\ N$~~

~~$R_{Av} = 3510.4288 - R_{Bv}$~~ ~~$R_{Av} = 3510.4288 - R_{Bv}$~~

~~$= 3510.4288 - 3811.5928$~~ ~~$R_{Av} = 278.1552\ N$~~

~~$R_{Av} = -301.164\ N$~~

~~$-868.9789 \times 250 + R_{Bv} \times 750$~~

$(868.9789 \times 250) - R_{Bv} \times 750 + 2641.449 \times 1000 = 0$

$\therefore \boxed{R_{Bv} = 3811.5916\ N}$

$R_{Av} = 3510.4288 - 3811.5916\ N$

$\therefore \boxed{R_{Av} = -301.1628\ N}$

Scanned with CamScanner

Bending moment at points

$M_{Dv} = 0$

$M_{Av} = 868.9789 \times 250 = 217.2447 \times 10^3$ N-mm

$M_{Bv} = 2641.4499 \times 250 \doteq 660.3624 \times 10^3$ N-mm

$M_{cv} = 0$

consider horizontal load diagram

~~$R_{AH} + R_{BV} = 1063.1616 - 23$~~

$R_{AH} + R_{BH} + 2387.5 - 1063.1616 = 0$

$R_{AH} + R_{BH} = 1063.1616 - 2387.5$

$\boxed{R_{AH} + R_{BH} = -1324.3384}$

consider moment at point A

$(-2387.5 \times 250) - R_{BH} \times 750 + 1063.1616 \times 1000 = 0$

$-R_{BH} \times 750 = 2387.5 \times 250 - 1063.1616 \times 1000$

$\cancel{-} R_{BH} = \cancel{-} \frac{466286.6}{750}$

$\therefore \boxed{R_{BH} = 621.71546 \text{ N}}$

$R_{AH} = -1324.3384 - R_{BH}$

$R_{AH} = -1324.3384 - 621.71546$

$\boxed{R_{AH} = -1946.053 \text{ N}}$

consider the Bending moment points

$M_{DH} = 0$

$M_{AH} = -2387.5 \times 250 = -596875$ N-mm

$M_{BH} = \cancel{621.7546 \times 250} = \cancel{155438.65\,N\text{-}mm}$ 1063.1616×250

$= 265.7904 \times 10^3\,N\text{-}mm$

$M_{CH} = 0$

Resultant Bending moment (M)

$$M_A = \sqrt{(M_{AV})^2 + (M_{AH})^2} = \sqrt{(217.2447 \times 10^3)^2 + (-596875)^2}$$

$$\boxed{M_A = 635.1810 \times 10^3\,N\text{-}mm}$$

$$M_B = \sqrt{(M_{BV})^2 + (M_{BH})^2} = \sqrt{(660.3624 \times 10^3)^2 + (265.7904 \times 10^3)^2}$$

$$\cancel{M_B = 678.4096 \times 10^3\,N\text{-}mm} \quad \boxed{M_B = 711.8448 \times 10^3\,N\text{-}mm}$$

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(0)^2 + (0)^2} = 0$$

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{0 + 0} = 0$$

The max. Bending moment occurs at point B

$$\therefore \boxed{M_{max} = 711.8448 \times 10^3\,N\text{-}mm}$$

According to ~~max. normal stress theory~~ ASME code shear

$$d^3 = \frac{16}{\pi \tau_{max}} \left(\sqrt{(C_m M)^2 + (C_t T)^2} \right) \rightarrow [\text{eq}^n\ 3.6(b),\ \text{Pg } 51]$$

where, assumes minor shocks

From table (3.1) Pg 56

$C_m = 1.75$

$C_t = 1.25$

$\therefore M_{max} = 711.8448 \times 10^3\,N\text{-}mm$

$\therefore T = 238.75 \times 10^3\,N\text{-}mm$

According to ASME standards

$\tau_{max} = 0.3 \times \sigma_y = 0.3 \times 310 = 93\,MPa$

$\tau_{max} = 0.18 \times \sigma_u = 0.18 \times 500 = 90\,MPa$

minimum of these two

$\tau_{max} = 90\,MPa$

Considering key way effect, $\tau_{max} = 0.75 \times 90$

$\therefore \boxed{\tau_{max} = 67.5\,MPa}$

$$d^3 = \frac{16}{\pi \times 67.5}\left[\sqrt{(1.75 \times 711.8448 \times 10^3)^2 + (1.25 \times 238.75 \times 10^3)^2}\right]$$

$$\boxed{d = 45.8918\,mm}$$

According to the standard shaft size in mm from table (3.5) pg 57

(or)

From the table (3.5) pg 57, st shaft size in mm

$$\therefore \boxed{d = 50mm}$$

6) A horizontal steel shaft supported on a bearing A at left end & B at right end carries two gears C & D located at distance 250mm & 400mm respectively from centre lines of left end & Right end bearings. The pitch diameter of gear C is 600mm & that of D is 200mm. The distance b/w the centre line of bearing is 2400 mm. The shaft transmitts 20kW at 120 rpm. The Power is delivered to the gear C & it is taken out at gear D in such a manner that the tooth pressure ~~FT & FTD of gear~~ F_{tC} & F_{tD} of gear C & D acts vertically downwards. The pressure angle is 20°. Find the diameter of the shaft if the working stresses are 100MPa in tension & 56MPa in shear. The gears C & D weighs 950N & 350N respectively. Taken C_m & C_T 1.5 & 1.2.

Sol^n

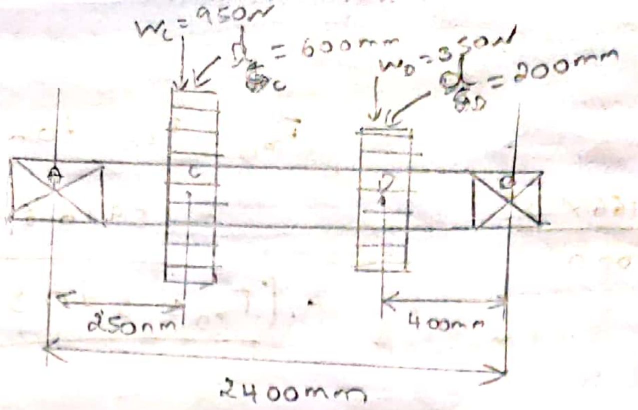
$d_{gC} = 600mm$ $\quad d = ?$

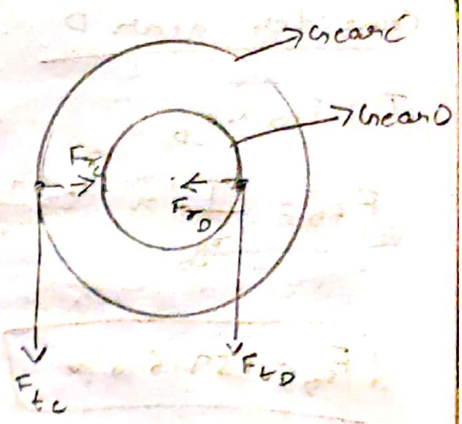
$d_{gD} = 200mm$ $\quad W_C = 950N$

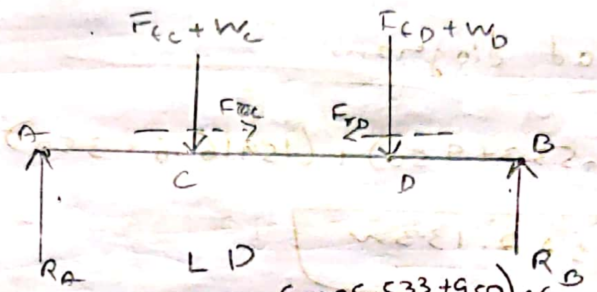
$\phi = 20°$ $\quad W_D = 350N$

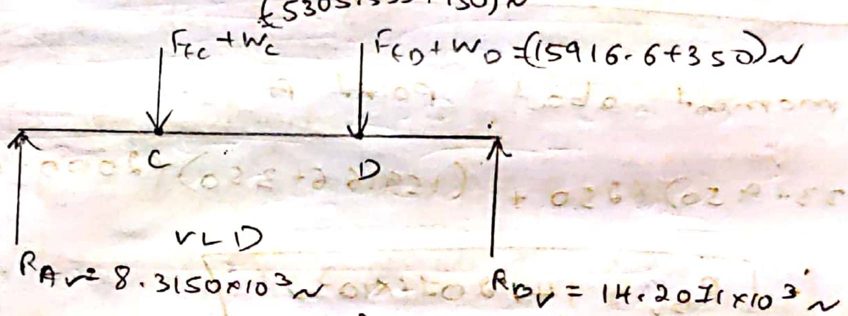
$P = 20kW$ $\quad C_m = 1.5$

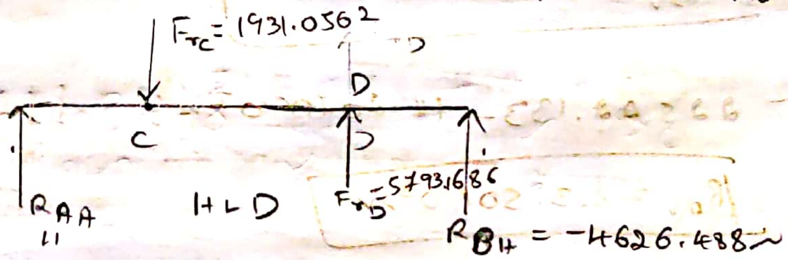
$N = 120rpm$ $\quad C_T = 1.2$











$R_{AH} = 764.3756N$

To find Torque $= \frac{9.55 \times 10^6 \times P}{n} = \frac{9.55 \times 10^6 \times 20}{120} = 1.5916 \times 10^6 N\text{-}mm$

consider gear C

To find F_{tc}

$T = F_{tc} \times r_g$

$F_{tc} = \frac{T}{r_g} = \frac{1.59166 \times 10^6}{300}$

$\therefore \boxed{F_{tc} = 5305.533N}$

$F_{rc} = F_{tc} \tan\phi$

$= 5305.533 \times \tan(20°)$

$\boxed{F_{rc} = 1931.0562N}$

Consider gear D

$T = F_{tD} \times r_{gD}$

$F_{tD} = \frac{T}{r_{gD}} = \frac{1.59166\times10^6}{1000}$

$\therefore \boxed{F_{tD} = 15916.6\ N}$

$F_{rD} = F_{tD} \times \tan\phi$

$= 15916.6 \times \tan(20^\circ)$

$\therefore \boxed{F_{rD} = 5793.16863\ N}$

Consider vertical load diagram

$R_{Av} + R_{Bv} = (5305.533 + 950) + (15916.6 + 350)$

$\therefore \boxed{R_{Av} + R_{Bv} = 22522.133\ N}$

Consider moment about point A

$(5305.533 + 950) \times 250 + (15916.6 + 350) \times 2000 - R_{Bv} \times 2400 = 0$

$\therefore \boxed{R_{Bv} = 14.2071\times10^3\ N}$

$R_{Av} = 22522.133 - 14.2071\times10^3$

$\therefore \boxed{R_{Av} = 8.3150\times10^3\ N}$

Consider bending moments about a points.

$M_{Av} = 0$

$M_{Bv} = 0$

$M_{Cv} = -8.3150\times10^3 \times 250$

$\boxed{M_{Cv} = -2.07875\times10^6\ N\text{-}mm}$

$M_{Dv} = -14.2071\times10^3 \times 400 = -5.68284\times10^6\ N\text{-}mm$

Consider horizontal load diagram

$R_{AH} + R_{BH} = 1931.0562 - 5793.1686$

$$\boxed{R_{AH} + R_{BH} = -3862.1124\,N}$$

consider moment about point A

$$7931.0562 \times 250 - (5793.16863 \times 2000) - R_{BH} \times 2400 = 0$$

$$-11.1035 \times 10^6 = R_{BH} \times 2400$$

$$\therefore \boxed{R_{BH} = -4626.488\,N}$$

$$R_{AH} = -3862.1124 - R_{BH}$$

$$R_{AH} = -3862.1124 - (-4626.488)$$

$$\boxed{R_{AH} = 764.3756\,N}$$

consider bending moment about points

$$M_{AH} = 0 \qquad M_{CH} = -764.3756 \times 250$$

$$M_{BH} = 0 \qquad \boxed{M_{CH} = -191.0939 \times 10^3\,N\text{-}mm}$$

$$M_{DH} = -R_{BH} \times 400 = -(-4626.488) \times 400$$

$$\boxed{M_{DH} = 1.8505952 \times 10^6\,N\text{-}mm}$$

Resultant Bending moments.

$$M_A = \sqrt{(M_{AV})^2 + (M_{AH})^2} = 0$$

$$M_B = \sqrt{(M_{BV})^2 + (M_{BH})^2} = 0$$

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(-2.07875 \times 10^6)^2 + (-191.0939 \times 10^3)^2}$$

$$\boxed{M_C = 2.08751 \times 10^6\,N\text{-}mm}$$

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(-5.68284 \times 10^6)^2 + (1.8505952 \times 10^6)^2}$$

$$\boxed{M_D = 5.97656 \times 10^6\,N\text{-}mm}$$

The max. Bending moment at point D

$$\boxed{M_{max} = 5.97656 \times 10^6 \; N\text{-}mm}$$

$\sigma_{max} = 100 MPa$ $\quad C_m = 1.5$

$\tau_{max} = 56 MPa$ $\quad$ & $C_t = 1.2$

According to ASME, max. normal stress

$$d^3 = \frac{16}{\pi \sigma_{max}} \left[C_m M + \sqrt{(C_m M)^2 + (C_t T)^2} \right]$$

$$= \frac{16}{\pi \times 100} \left[(1.5 \times 5.97656 \times 10^6) + \sqrt{(1.5 \times 5.97656 \times 10^6)^2 + (1.2 \times 1.59166 \times 10^6)^2} \right]$$

$$\therefore \boxed{d = 97.37849 \; mm}$$

According to ASME max. shear stress

$$d^3 = \frac{16}{\pi \tau_{max}} \left[\sqrt{(C_m M)^2 + (C_t T)^2} \right]$$

$$= \frac{16}{\pi \times 56} \left[\sqrt{(1.5 \times 5.97656 \times 10^6)^2 + (1.2 \times 1.59166 \times 10^6)^2} \right]$$

$$\therefore \boxed{d = 94.11410745 \; mm}$$

From the table 3.5 Pg 57

Standard shaft size

$D = 100 \; mm$

17 A power transmission shaft 1800mm is supported (bearing) at two points A & B. A is at 300mm from left extreme end & B is at Right extreme end. The power of 30 kW is received at 600rpm through a gear drive located at the left extreme end of the shaft. This gear has fixed diameter of 300mm & weighs 700N. That drive gear is placed exacting behind. This 18 kW of this Power is given out through a drive pulley located at a distance of 600mm from the left support. The pulley mounted on the shaft as a diameter of shaft 400mm & weighs 1000N, the belt drive is directed towards the absorber below horizontal & inclined at an angle of 45° to it. The ratio of belt tension is 3. The remaining power is given out through a gear derive at a distance of 400mm from the right support. The gear has a fixed diameter of 200mm & weighs 500N. Find suitable diameter of the shaft taking allowable stress has 120MPa in tension & 75MPa in shear.

Sol:

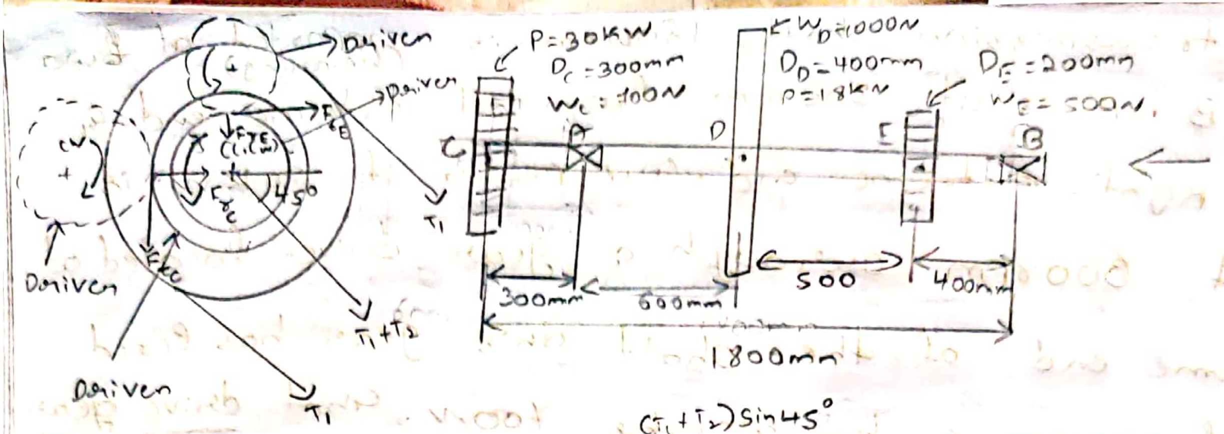
$P = 30kW$

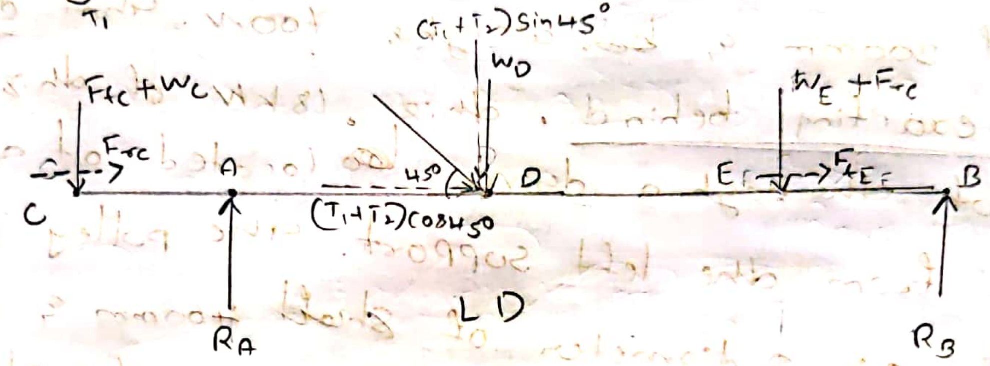
$n = 600rpm$

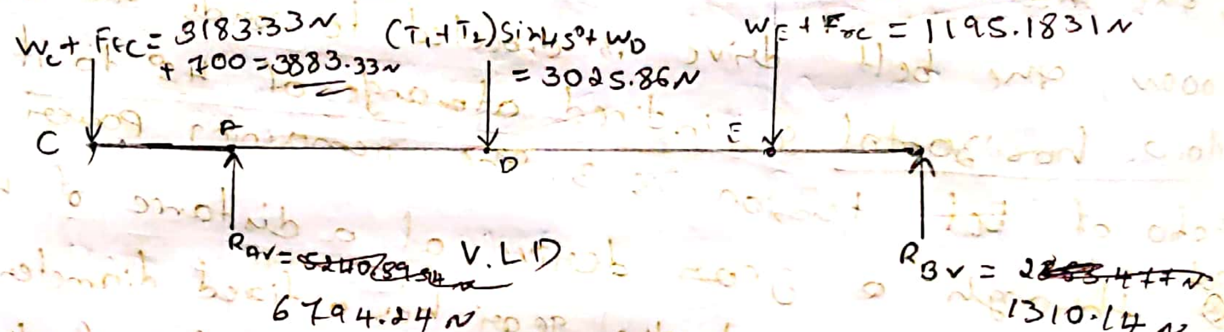
$\frac{T_1}{T_2} = 3$

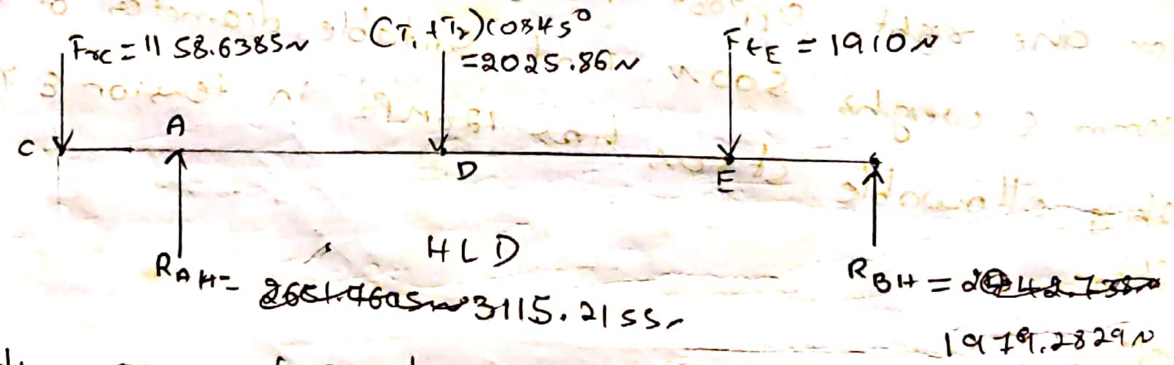
$\sigma_{max} = 120MPa$

$\tau_{max} = 75MPa$









Consider the gear at point c

$P = 30\,kW$

$n = 600\,rpm$

$$T = \frac{9.55\times10^6 \times P}{n} = \frac{9.55\times10^6 \times 30}{600}$$

$$\therefore \boxed{T = 477.5\times10^3\ N\text{-}mm}$$

wkT Torque, $T = F_{tc} \times r_{gc}$

$$F_{tc} = \frac{T}{r_{gc}} = \frac{477.5\times10^3}{150}$$

$$\boxed{F_{tc} = 3183.33\,N}$$

Scanned with CamScanner

WKT $F_{rC} = F_{tC} \times \tan\phi = 3183.33 \times \tan(20^\circ)$

$\boxed{F_{rC} = 1158.6385\ N}$

Consider the pulley at D $\quad P = 18\ kN$

Torque, $T = \dfrac{9.55\times10^6 \times P}{n} = \dfrac{9.55\times10^6 \times 18}{600}$

$\boxed{T = 286.5\times10^3\ N\text{-}mm}$

WKT $T = (T_1 - T_2) r_{P_D}$ $\quad \dfrac{T_1}{T_2} = 3$

$T = (3T_2 - T_2)200$ $\quad T_1 = 3T_2$

$286.5\times10^3 = 2T_2 \times 200$ $\quad T_1 = 3\times716.25\ N\text{-}mm$

$\boxed{T_2 = 716.25\ N}$ $\quad \boxed{T_1 = 2148.75\ N}$

$(T_1 + T_2) = 2865\ N\text{-}mm$

$(T_1+T_2)\sin45^\circ = 2025.860\ N$

$(T_1+T_2)\cos45^\circ = 2025.86\ N$

Total vertical force

$(T_1+T_2)\sin45^\circ + W_D = 2025.86 + 1000$

$= 3025.86\ N$

Consider the gear E

$P_E = P_C - P_P = 30 - 18 = 12\ kN$

$T = \dfrac{9.55\times10^6 \times P}{n} = \dfrac{9.55\times10^6\times12}{600} = 191\times10^3\ N\text{-}mm$

Torque, $T = F_{tE} \times r_{gE}$

$F_{tE} = \dfrac{T}{r_{gE}} = \dfrac{191\times10^3}{100}$

$\therefore \boxed{F_{tE} = 1910\ N}$

$F_{rE} = F_{tE} \times \tan\phi$

$= 1910 \times \tan(20^\circ)$

$\therefore \boxed{F_{rE} = 695.1831\ N}$

Consider vertical load diagram

$R_{Av} + R_{Bv} = 3883.33 + 3025.86 + 1195.1831$

$$\boxed{R_{Av} + R_{Bv} = 8104.3731\ N}$$

Consider moment about point A

$-(3883.33 \times 300) + (3025.86 \times 600) + (1195.1831 \times 1100)$
$-(R_{Bv} \times 1500) = 0$

$\cancel{R_{Bv} = 2863.477}$ $R_{Bv} = 1310.1453\ N$

$\cancel{R_{Av} = 8104.3731 - R_{Bv} = 8104.3731 - 2863.477}$

$\cancel{R_{Av} = 5240.8934}$ 6794.22 N

$R_{Av} = 6794.22\ N$

Consider bending moments at points.

$M_{Cv} = 0$

$M_{Av} = 3883.33 \times 300 = 1.1649 \times 10^6\ N\text{-}mm$

$M_{Dv} = 1195.1831 \times 500$ ~~ - (2863.477 × 900)~~ $-(1310.4455 \times 900)$ $= -581.535 \times 10^3$

$M_{Ev} = -$ ~~2863.477 × 400 = -1.1453908 × 10^6 N-mm~~ $= -524.058 \times 10^3$

$M_{Bv} = 0$

Consider Horizontal load diagram

$R_{AH} + R_{BH} = 1158.6385 + 2025.86 + 1910$

$$\boxed{R_{AH} + R_{BH} = 5094.4985\ N}$$

Consider moment about point B

$(1158.6385 \times 1800) + (2025.86 \times 900) + (1910 \times 400) - (R_{AH} \times 1500) = 0$

~~$R_{AH} = 2442.438\ N$~~ $\boxed{R_{AH} = 3115.2155\ N}$

$$\cancel{R_{AH} = 5094.4985 - R_{BH}}$$

$$\cancel{= 5094.4985 - 2442.738}$$

$$\cancel{\boxed{R_{AH} = 2651.7605 \, N}}$$

$$R_{BH} = 5094.4985 - R_{AH}$$

$$R_{BH} = 1979.282967 \, N$$

consider bending moments at points

$$M_{CH} = 0$$

$$M_{AH} = (1158.6385 \times 300) = 347.591 \times 10^3 \, N\text{-}mm$$

$$\cancel{M_{DH} = (1910 \times 500) - (\underset{2442.738}{3115.2155} \times 900) = -1.84869 \times 10^6 \, N\text{-}mm}$$

$$\cancel{M_{EH} = (2442.738 \times 400) = 977.0952 \times 10^3 \, N\text{-}mm}$$

$$M_{BH} = 0$$

~~Resultant bending moments~~

$$M_{DH} = (1910 \times 500) - (1979.2829 \times 900) = -826.3546 \times 10^3 \, N\text{-}mm$$

$$M_{EH} = -(1979.2829 \times 400) = -791.713 \times 10^3 \, N\text{-}mm$$

$$M_{BH} = 0$$

Resultant bending moments

$$M_A = \sqrt{(M_{AV})^2 + (M_{AH})^2} = \sqrt{(1.1649 \times 10^6)^2 + (347.591 \times 10^3)^2}$$

$$\boxed{M_A = 1.2156 \times 10^6 \, N\text{-}mm}$$

$$M_B = 0$$

$$M_C = 0$$

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(-581.535 \times 10^3)^2 + (-826.3546 \times 10^3)^2}$$

$$\therefore \boxed{M_D = 1.01046 \times 10^6 \, N\text{-}mm}$$

$$M_E = \sqrt{(M_{EV})^2 + (M_{EH})^2} = \sqrt{(\cancel{-1.14539\times10^6})^2 + (-791.713\times10^3)^2}$$
$$(-524.058\times10^3)^2$$

$$\boxed{\cancel{M_E = 1.39273\times10^6\ N\text{-}mm}}\quad 949.445\times10^3\ N\text{-}mm$$

of A

$$\boxed{\cancel{M_{max} = 1.3923\times10^6\ N\text{-}mm}}\quad M_{max} = 1.2156\times10^6\ N\text{-}mm$$

$$\boxed{T_{max} = 477.5\times10^3\ N\text{-}mm}$$

$$d^3 = \frac{16}{\pi\sigma_{max}}\left[C_m M + \sqrt{(C_m M)^2 + (C_t T)^2}\right]$$

$C_m = 1.75$

$C_t = 1.25$

$$d^3 = \frac{16}{\pi\times120}\left[(1.75\times1.2156\times10^6) + \sqrt{(1.75\times1.2156\times10^6)^2 + ((1.25)\times477.5\times10^3)^2}\right]$$

$$\boxed{\cancel{d = 56.497\ mm}}\quad \boxed{d = 56.88\ mm}$$

$$d^3 = \frac{16}{\pi\tau_{max}}\left[\sqrt{(C_m M)^2 + (C_t T)^2}\right]$$

$$d^3 = \frac{16}{\pi\times47.5}\left[\sqrt{(1.75\times1.2156\times10^6)^2 + (1.25\times477.5\times10^3)^2}\right]$$

$$d\ \boxed{\cancel{d = 52.885\ mm}}\quad \cancel{d = 55.434\ mm}$$

$$d = 53.1370\ mm$$

Scanned with CamScanner

8) A hoisting drum of 500mm diameter is keyed onto a shaft & it interested to lift a load of 20kN at a velocity of 31.4 m/min the shaft is supported on two bearings & carries a gear of 400mm diameter, over-hanging the nearest bearing by 200mm (i.e. 200mm to the right of right bearing) the gear ratio is 12:1 determine the power & revolutions per minute of the motor required the gear drive has an efficiency of 90%. determine the diameter of the shaft for the hoisting drum assuming that the material of the shaft has an allowable shear stress of 16MPa the distance b/w the bearing is 1000mm the pressure angle is 20°. for suddenly applied load with minor shock the fatigue factor to be applied to the computed bending moment & the numerical combined shock & fatigue factor to be applied to the torsional moment are 2 & 1.3 bending moment (C_m & C_T) sketch the relevant diagram.

Sol:

$d_h = 500mm$

$W = 20kN$

$v = 31.4 m/min = \cancel{\frac{31.4}{60} m/sec = 0.523 m/sec}$

$i = 12:1$

$\eta = 90\% = 0.9$

$N = ?$

$P = ?$

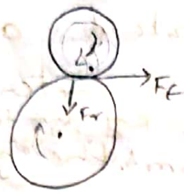
$D = ?$

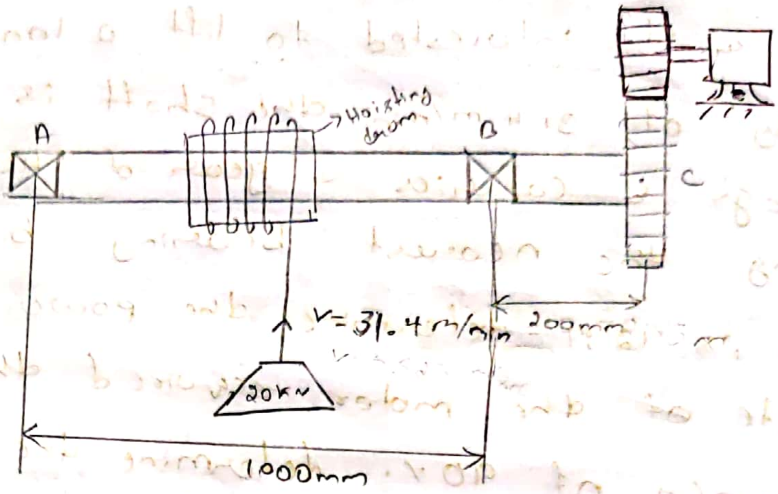
$\tau_{max} = 16MPa$

$\phi = 20° \; [\varphi = 20°]$

$C_m = 2$

$C_t = 1.3$





Torque on hoisting drum = load to be lifted × Radius of hoisting drum

$$T = 20\times10^3\times250$$

$$\boxed{T = 5\times10^6\ N\text{-}mm}$$

$$\therefore \boxed{T = 5\times10^3\ N\text{-}m}$$

Angular speed of hoisting drum, $\omega = \frac{v}{r}$ [$\because v = \omega r$]

$$\omega = \frac{31.4\ m/min}{0.25\ m}$$

$$\therefore \boxed{\omega = 125.6\ rad/min}$$

Angular speed of motor = (12×125.6) [Gear ratio = 12:1]

$$\boxed{\omega = 1507.2\ rad/min}$$

$$i = \frac{n_1}{n_2}$$

$$n_1 = i\times n_2$$

$$n_1 = 12 n_2$$

Speed of motor $\omega = 2\pi N$

$$N = \frac{\omega}{2\pi} = \frac{1507.2}{2\times\pi}$$

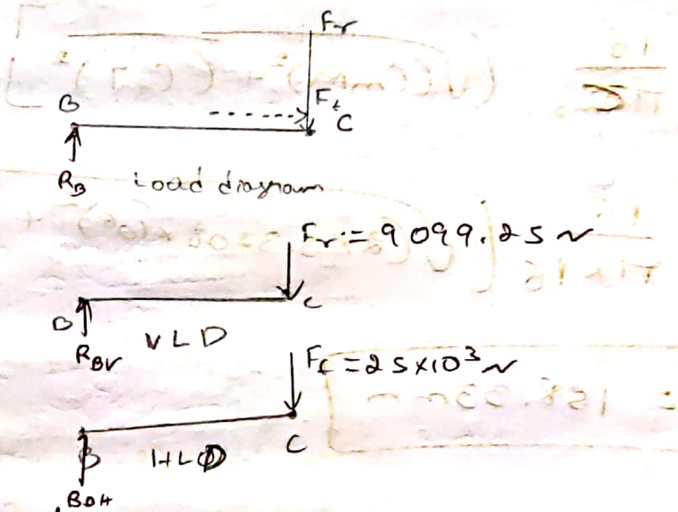
$$\therefore \boxed{N = 239.87\ rpm}$$

power of electric motor,

$$P = \frac{T \times \omega}{60 \times 1000 \times \eta}$$

$$P = \frac{5 \times 10^3 \times 125.6}{60 \times 1000 \times 0.9}$$

$$\therefore P = \underline{11.629\,kW}$$



consider the gear at point C

$T = F_t\, r_g$

$$\frac{5 \times 10^6\,N\text{-}mm}{200} = F_t$$

$$\boxed{F_t = 25 \times 10^3\,N}$$

$F_r = F_t \tan\phi$

$= 25 \times 10^3 \tan(20^\circ)$

$$\therefore \boxed{F_r = 9099.25\,N}$$

consider vertical load diagram.

Bending moment

$M_{BV} = (9099.25 \times 200) = 1.819851 \times 10^6\,N\text{-}mm$

$M_{CV} = 0$

Consider horizontal load diagram.

Bending moment

$M_{BH} = (25 \times 10^3) \times 200 = 5 \times 10^6\,N\text{-}mm$

$M_{CH} = 0$

Resultant bending moment

$$M_B = \sqrt{(M_{Bv})^2 + (M_{BH})^2} = \sqrt{(1.8198\times10^6)^2 + (5\times10^6)^2}$$

$$\boxed{M_B = 5.3208\times10^6\ N\text{-}mm}$$

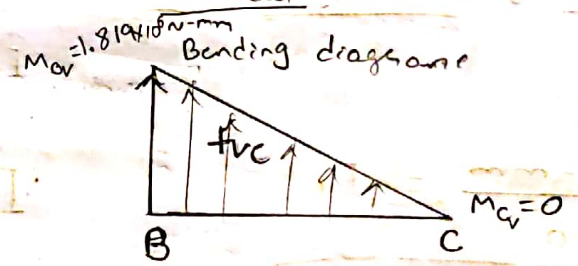
According to ASME code for max. ~~normal~~ shear stress theory

$$d^3 = \frac{16}{\pi\tau_{max}}\left[\sqrt{(C_m M)^2 + (C_t T)^2}\right]$$

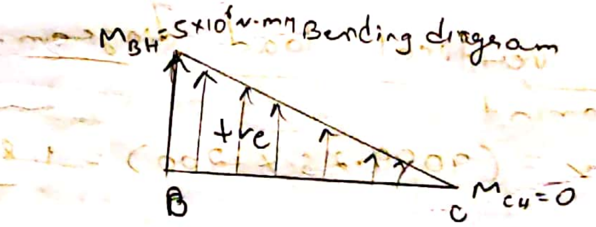
$$= \frac{16}{\pi\times16}\left[\sqrt{(2\times5.3208\times10^6)^2 + (1.3\times5\times10^6)^2}\right]$$

$$\therefore \boxed{d = 158.33mm}$$

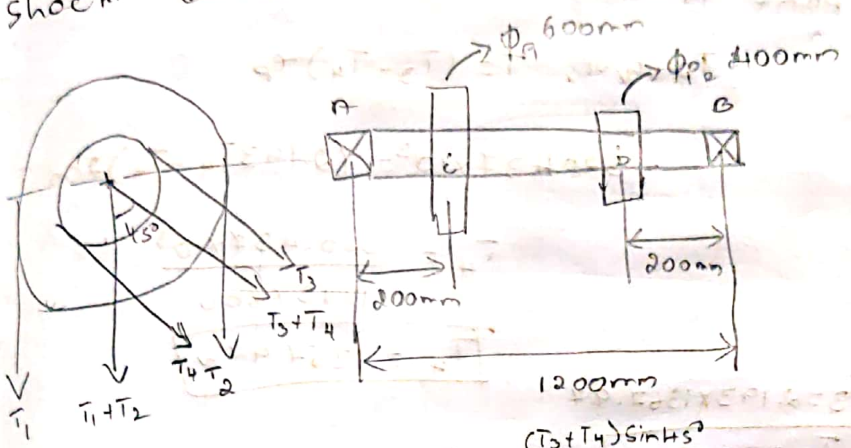
Vertical L.D

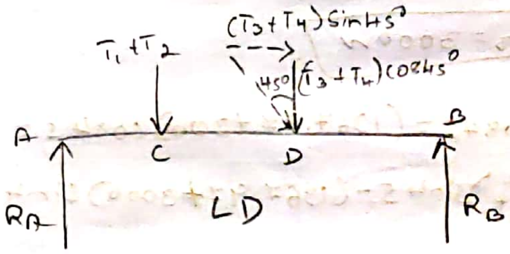


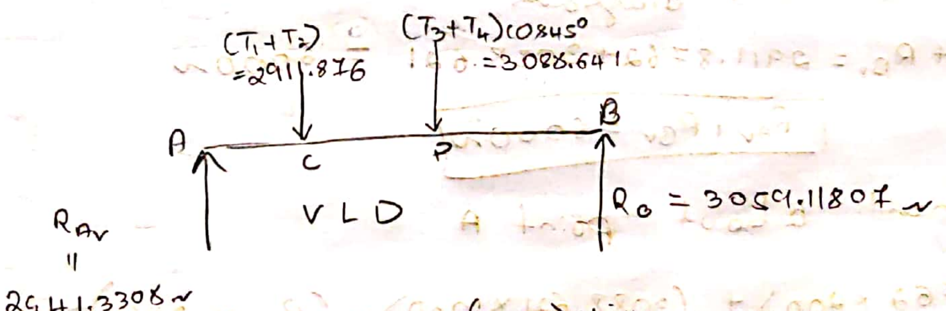
Horizontal L.D

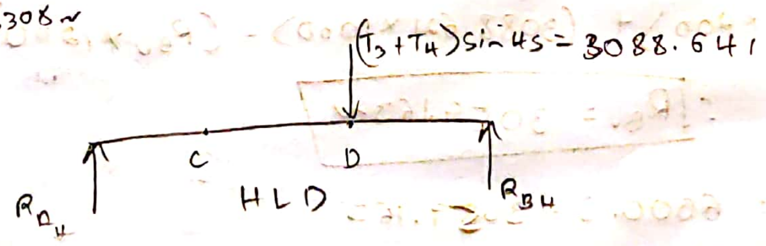


Q) A hollow shaft is supported by two bearing placed 1.2m apart a 600mm diameter pulley is mounted at a distance of 200mm to the right of left hand bearing & this drives a pulley directly below it with a help of a belt having maximum tension of 2kN, & another pulley of 400mm diameter is placed 200mm to the left of Right bearing & it is driven with the help of motor & belt which is placed at 45° to vertical downwards towards the observer the angle of contact for both pulley is 180° & Co-efficient friction is 0.25. Determine the suitable diameter for hollow shaft having diameter ratio of 0.5. The allowable tensile & shear stress are 63MPa & 42MPa respectively & assume heavy shock condition.









Data

$T_1 = 2kN$

$\theta = 180°$

$\mu = 0.25$

$k = 0.5$

Consider pulley at C

wkT $\frac{T_1}{T_2} = e^{\mu\theta}$

$$T_2 = \frac{T_1}{e^{\mu\theta}} = \frac{2\times10^3}{e^{(0.25\times180\times\pi/180)}}$$

$$\therefore \boxed{T_2 = 911.87662\text{N-mm}}$$

Torque $= (T_1 - T_2) r_p$

$T = (2000 - 911.876) \times 300$

$\underline{T = 326.437 \times 10^3 \ N\text{-}mm}$

consider the pulley at point D

$\frac{T_3}{T_4} = e^{\mu\theta}$

$T_3 = T_4 \times e^{\mu\theta}$

$T_3 = e^{(0.25 \times \pi/180 \times 180)} T_4$

$\boxed{T_3 = 2.193 T_4}$

Torque, $T = (T_3 - T_4) r_p$

$326.437 \times 10^3 = (2.193 T_4 - T_4) 200$

$T_4 = \frac{326.437 \times 10^3}{1.193 \times 200}$

$\boxed{T_4 = 1367.97 N}$

$T_3 = 2.193 \times 1367.97$

$\therefore \boxed{T_3 = 3000 N}$

Total vertical force, $= (T_3 + T_4) \cos 45 = (1367.97 + 3000) \cos 45 = 3088.621 N$

Total horizontal force $= (T_3 + T_4) \sin 45 = (1367.97 + 3000) \sin 45 = \underline{3088.621 N}$

consider vertical load diagram

$R_{Av} + R_{Bv} = 2911.8762 + 3088.621 = 6000 N$

$\boxed{R_{Av} + R_{Bv} = 6000 N}$

Taking moment about point A

$(2911.8762 \times 200) + (3088.621 \times 1000) - (R_{Bv} \times 1200) = 0$

$\therefore \boxed{R_{Bv} = 3059.163 N}$

$R_{Av} = 6000.5 - 3059.163$

$\boxed{R_{Av} = 2941.337 N}$

Taking moment at point,

$M_{Av} = 0$

$M_{Bv} = 0$

$M_{Cv} = (-2941.337 \times 200) = -588.267 \times 10^3 \ N\text{-}mm$

$M_{Dv} = -(3059.163 \times 200) = -611.8326 \times 10^3 \ N\text{-}mm$

consider the horizontal load diagram

$R_{AH} + R_{BH} = 3088.62$

Taking moment about point A

$$(3088.621 \times 1000) - (R_{BH} \times 1200) = 0$$

$$R_{BH} = \frac{3088.621 \times 1000}{1200}$$

$$\boxed{R_{BH} = 2573.850 \text{ N}}$$

Taking moment at points.

$M_{AH} = 0$

M